

Schuch–Pérez-García–Cirac SPT Interpolation: Upper-Range Correction

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Verdict: local-correction (resolved).

1 Source construction

Section II.F of arXiv:1010.3732 connects two symmetric injective matrix product states after passing to their isometric forms. Let their bond dimensions be D_0 and D_1 . The two bond spaces are embedded as consecutive summands of a space of dimension $D_0 + D_1$, and the interpolation is intended to move between maximally entangled vectors supported on those two summands. The source display in `Papers/1010.3732/paper_v3.tex`, lines 900–903, reads

$$|\omega(\gamma)\rangle = (1 - \gamma) \sum_{i=1}^{D_0} |i, i\rangle + \gamma \sum_{i=D_0+1}^{D_1} |i, i\rangle, \quad (\text{source display})$$

and immediately says that this state has bond dimension $D_0 + D_1$.

2 Issue and correction

The second summand must occupy the D_1 coordinates following the first D_0 coordinates. Its index set is therefore

$$\{D_0 + 1, \dots, D_0 + D_1\}, \quad (1)$$

not $\{D_0 + 1, \dots, D_1\}$. The corrected interpolation is

$$|\omega(\gamma)\rangle = (1 - \gamma) \sum_{i=1}^{D_0} |i, i\rangle + \gamma \sum_{i=D_0+1}^{D_0+D_1} |i, i\rangle. \quad (\text{corrected interpolation})$$

At $\gamma = 0$ this is the maximally entangled vector on the first bond summand, while at $\gamma = 1$ it is the maximally entangled vector on the second bond summand. The corrected upper limit also agrees with the stated ambient bond dimension $D_0 + D_1$. The source upper limit can otherwise make the second sum empty, for example when $D_0 \geq D_1$.

3 Blueprint and formal status

The corrected display occurs in the proof of the blueprint theorem `thm:spt_classification` in `blueprint/src/chapter/ch12_symmetry_spt.tex`, where an inline `Localfix` comment points to this note. The nearby separation argument at source lines 929–953 is explicitly restricted to smooth MPS paths. The blueprint theorem is correspondingly restricted to symmetric MPS paths and their parent Hamiltonians; it does not claim invariance under arbitrary gapped Hamiltonian paths.

The formal predicate `MPSTensor.IsSameSPTPhase` records equality of the virtual cocycle labels, not the interpolation. The blueprint classification theorem is marked `notready`; no Lean declaration implements this path or relies on the corrected summation range.

4 Verdict

The replacement of D_1 by $D_0 + D_1$ is a local source correction. It restores the intended direct-sum coordinates and is the version used in the blueprint. The MPS/parent-Hamiltonian path restriction is a scope clarification matching the source proof.