

Uniform native-closure regular PEPS renormalization

2026-10-05

Verdict: scope-restriction (open).

1 Source and proved specialization

The source is Schuch, Cirac, and Pérez-García, arXiv:1001.3807v3, Definition 5.6, Theorem 5.9 and Observation 6.6, local source lines 1515–1525, 1582–1621 and 1888–1909.

Let G be an arbitrary finite group and A a homogeneous regular G -isometric four-leg tensor with an arbitrary finite physical alphabet. For coarse periods $w, h \geq 3$, one isometry I on the full product block physical support satisfies, for every native closure pair (g_0, h_0) ,

$$I\Psi_f(g_0, h_0) = R\Psi_c(g_0, h_0) \otimes \Omega, \quad I\hat{\Psi}_f(g_0, h_0) = \hat{\Psi}_c(g_0, h_0) \otimes \Omega.$$

Here the fine periods are $2w, 2h$, the coarse tensor is literally A , Ω is a fixed unit Bell product, and

$$R = \left(\prod_v \frac{\sqrt{c_{B,v}}}{\sqrt{c_{C,v}}} \right) (\sqrt{|G|})^{|E(\Gamma)|} > 0.$$

Neither I , its local physical matrices, nor R depends on the closure. The factors are derived from the G -isometry of the actual four-site block and of the original tensor with surplus identity registers. The previous four-site Gram result retains its internal-cycle group-order factor. Each native closure is proved nonzero rather than assumed nonzero.

The horizontal seam carries h_0 on the receiving left legs; the vertical seam carries g_0 in the native downward orientation. Reversing an arrow to express it as an ordered graph edge transposes its inserted matrix and therefore inverts the regular group label. Both paired bond labels receive the same insertion before the relative-coordinate change. The identity $(ua)^{-1}(ub) = a^{-1}b$ leaves the surplus label unchanged.

2 Coherent vectors and ground-state scope

For arbitrary complex coefficients z_{g_0, h_0} , the same map gives

$$I \sum_{g_0, h_0} z_{g_0, h_0} \Psi_f(g_0, h_0) = R \left(\sum_{g_0, h_0} z_{g_0, h_0} \Psi_c(g_0, h_0) \right) \otimes \Omega.$$

Fine and coarse sums vanish simultaneously, and their nonzero normalized vectors satisfy the corresponding exact identity. Thus relative coefficients and phases are preserved. Restricting the coefficients to commuting pairs proves this assertion for arbitrary coherent vectors in the commuting-closure span. The algebraic result also covers noncommuting pairs but makes no parent-ground-space claim for them. Identification with a particular parent kernel uses the separate parent-ground-space theorem; it is not inferred from noncommuting closure nonvanishing.

3 Remaining restrictions

This result removes the identity-closure restriction of the earlier periodic original-tensor fixed-point specialization. It preserves its homogeneous, finite regular, physical-support scope. The native geometric regrouping includes all positive coarse periods, but the physical-support and Bell-separation argument requires the simple coarse graph and therefore periods at least three. Extending it to bond-indexed loops and parallel bonds is still required for periods one and two.

All original physical registers are retained. An ambient-space unitary extension is not asserted; unused physical directions are permitted. Open or other boundary conditions, nonregular representations, approximate schemes, and a general parent-Hamiltonian renormalization identity remain outside the theorem. These are the remaining qualifications to the source's unrestricted fixed-point formulation.

4 Declarations

The scalar-one native geometric identity is `TNLean.PEPS.torusGClosure_leftRegular_eq_twoByTwoB` `locked`. The native-to-bundled contraction comparison is `TNLean.PEPS.torusGClosure_leftRegular_eq_twoByTwoBundledGraphSite`. Uniform inserted physical support transport is `TNLean.PEPS.exists_graphGIsometricInsertedPhysicalSupportTransport`. The exact original-tensor surplus factorization is `TNLean.PEPS.graphInsertedBondNetwork_regularGraphSurplusSite`. The uniform capstone, including all coherent sums, is `TNLean.PEPS.exists_regularTwoByTwoTwistedTensorIsometry`.