

Quantum-double Hamiltonian terms: normalization and local scope

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Verdict: local-correction (resolved).

1 Source and normalization correction

Schuch, Cirac, and Pérez-García, arXiv:1001.3807v3, `Papers/1001.3807/paper_v3.tex`, lines 2861–2877, gives the product-one diagonal plaquette term and a coherent group-action term. The latter is printed as $I - \sum_{u \in G} V_u$, with the basis indices implicit. The action convention is $L_u|z\rangle = |uz\rangle$ and $R_u|z\rangle = |zu^{-1}\rangle$, according to the arrows.

For this permutation representation, put $S = \sum_u V_u$. Then

$$S^\dagger = S, \quad S^2 = |G|S, \quad (1)$$

$$A = |G|^{-1}S, \quad A^\dagger = A, \quad A^2 = A. \quad (2)$$

On every invariant vector, $(I - S)\psi = (1 - |G|)\psi$. Except for the trivial group, the printed expression therefore does not annihilate the intended invariant state. The projector-complement Hamiltonian term is $h = I - A$. This corrects the group average; it does not rescale the color-difference tensor or assume that its state is normalized.

2 Exact tensor and orientation

The tensor retained from source lines 2896–2916 is

$$K(p, q, r, s) = |pq^{-1}, qr^{-1}, rs^{-1}, sp^{-1}\rangle. \quad (3)$$

Equation (7.10) and its footnote, source lines 2896–2908, explicitly prescribe clockwise rotation. We label its virtual positions north, east, south, west by p, q, r, s , and its physical positions northeast, southeast, southwest, northwest by a, b, c, d . The available `figs6/tc-lattice.pdf` and `figs6/tc-blocked-lattice.pdf` show which sites and plaquettes are blocked and which color bonds are shared. These earlier drawings are used only for incidence and blocking. The physical arrows and nonabelian multiplication order used here are fixed

by Equation (7.10) and its clockwise footnote; no literal agreement with each physical arrow of the earlier drawings is asserted or needed. The unavailable `figs6/renorm-as-diff-doubles` figure is not treated as inspected evidence; the coordinate assignments above and all nonabelian products are stated explicitly.

At an east–west bond the shared color is $q_L = s_R = x$. Replacing x by ux acts on the actual eight physical spins as

$$(a, b, c, d; e, f, g, h) \mapsto (au^{-1}, ub, c, d; e, f, gu^{-1}, uh). \quad (4)$$

The hole uses the ordered spin tuple $(NW.b, SW.a, SE.d, NE.c)$. Shared colors give these four spins as $(xy^{-1}, yz^{-1}, zw^{-1}, wx^{-1})$, whose product is one. Neither identity assumes an abelian group.

3 Local assertions and remaining obligations

The local tensor image is the entire span of product-one computational basis vectors. When $abcd = 1$, an explicit preimage is

$$F(1, a^{-1}, (ab)^{-1}, (abc)^{-1}) = (a, b, c, d). \quad (5)$$

Thus the local flatness projector is the actual range projector of K , rather than an assumed tensor-image condition.

The east–west group average fixes the actual open two-tensor contraction before every choice of the six exterior colors: reindex its shared-color sum by $x \mapsto ux$. The diagonal four-block plaquette projector fixes the actual open four-tensor contraction before every choice of the eight exterior colors, by the displayed telescoping product. Their complements annihilate the respective contractions.

More precisely, let C be the two-block contraction map and $P_{00} = P_0 \otimes P_0$ the two local flatness constraints. Then $P_{00}A_e = A_eP_{00}$ and

$$\text{ran } C = \text{ran}(A_eP_{00}). \quad (6)$$

For a product-one physical pair $\xi = (a, b, c, d; e, f, g, h)$, set

$$\beta(\xi) = (a, b^{-1}, (bc)^{-1}; efg, fg, g). \quad (7)$$

These boundary colors reconstruct ξ at shared color one. Averaging that physical basis vector gives exactly $|G|^{-1}$ times the corresponding contraction column. This supplies the reverse range inclusion; tensor-image membership is not assumed.

The four-block hole projector also commutes with the coherent action on its northern bond and with that bond’s normalized average. The hole holonomy is conjugated by u , so the product-one condition is preserved even on the full ambient physical space.

Source lines 2918–2923 describe the three families of blocked Hamiltonian terms. The present assertions are a local part of that description. The following statements require further arguments:

1. a placement of all three families on a specified full lattice, with every bond orientation and boundary convention accounted for;
2. commutation of all placed terms and a full common-kernel identification, including the appropriate topological sectors;
3. a comparison of that explicit common kernel with the existing PEPS parent Hamiltonian, rather than merely local annihilation;
4. the actual unblocked arbitrary-group T -to- K physical renormalization, with its redundant factors and normalization.

The local physical relabelling between the review's dual tensor and the clockwise tensor does not construct the last operation. Generic parent classification results do not establish the second or third item unless their local kernels are identified with the explicit Hamiltonian here.