

Finite-Group Commutants: The Unital Qualification

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Verdict: local-correction (resolved).

1 The source and its necessary qualification

Schuch, Cirac, and Pérez-García state that every complex matrix C^* -subalgebra is the commutant of a unitary representation of a finite group [1, Theorem 4.1]. Their proof uses

$$\mathcal{D} \cong \bigoplus_i I_{d_i} \otimes M_{m_i}(\mathbb{C}), \quad (1)$$

with no zero summand. Source traceability is `Papers/1001.3807/paper_v3.tex`, lines 852–886. Earlier, however, the source defines a C^* -algebra simply as a linear matrix space closed under multiplication and adjoint, at lines 561–563. That wording alone does not require the ambient identity to belong to $\mathcal{D}$.

The ambient identity belongs to every commutant. Consequently the claim requires $I_D \in \mathcal{D}$. For example, $\{\text{diag}(z, 0) : z \in \mathbb{C}\} \subset M_2(\mathbb{C})$ is closed under multiplication and adjoint, but cannot be a commutant in $M_2(\mathbb{C})$. This explains why the absence of a zero summand in (1) matters.

2 The formal statement and the supported interpretation

The library theorem is stated for a unital complex star subalgebra, so membership of the ambient identity is part of the mathematical object. It constructs the finite group and its unitary representation from the subalgebra, and assumes no block decomposition or irreducible representations as additional data. Its blueprint entry explicitly states the unital qualification and is the unital version of Theorem 4.1, rather than the unrestricted reading of the earlier wording. This is a local correction to the source statement.

For a subalgebra whose identity is a proper projection p , the appropriate ambient algebra is $pM_D(\mathbb{C})p$, acting on $p\mathbb{C}^D$. There the subalgebra is unital,

and the corrected commutant theorem applies. In the matrix-product-state application, passing to this support removes unused bond directions. No claim about the commutant in the original larger matrix algebra follows.

References

- [1] Norbert Schuch, J. Ignacio Cirac, and David Pérez-García. PEPS as ground states: Degeneracy and topology. *Annals of Physics*, 325(10):2153–2192, 2010.