

SPT Fixed Points from a Supplied Projective Representation

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Verdict: scope-restriction (resolved).

1 What the source states

In the paragraph “Symmetry protected topological order” of [CPGSV21], a representative MPS with zero correlation length is constructed “starting from any solution of the 1- and 2-cocycle condition”: the input is a 2-cocycle ω and a 1-cocycle ϕ of a finite group G alone.¹

2 What is formalized

The dimer tensor $A^{(a,b)} = D^{-1/2}|a\rangle\langle b|$ depends on the bond dimension D alone, and its transfer, idempotence, injectivity, normality and linear-independence results involve no representation; the correction of the printed tensor is recorded in [con26a]. The symmetry results take as input a projective representation ρ of G on $\mathbb{C}^D$ whose factor system is ω , together with a character ϕ : the physical action $\phi(g) W_g^T \otimes W_g^{-1}$, $W_g = \rho(g^{-1})$, its twist identity, the on-site symmetry up to the character, and the realization of the class of ω (`MPSTensor.sptFixedPointAction`, `MPSTensor.twistedTensor_sptFixedPointTensor`, `MPSTensor.sptFixedPointTensor_isOnSiteSymmetricUpToCharacter`, `MPSTensor.cohomologousTo_of_sptFixedPointTensor`, `MPSTensor.exists_virtualRep_sptFixedPointTensor`). These results hold for any group. For a finite group, the next section supplies such a representation for every 2-cocycle class. Its factor system is a representative of the class of ω , not ω itself. Composed with these results, it gives the source’s construction from the cocycle alone.

¹Source traceability: `Papers/2011.12127/TN-Review-main.tex`, lines 1147–1157.

3 Resolution for finite groups

For finite G every 2-cocycle class arises. The twisted left regular matrices

$$L_g^\tau |h\rangle = \tau(g, h)^{-1} |gh\rangle \quad (1)$$

on $\mathbb{C}^G$, built from a circle-valued 2-cocycle τ , are unitary and satisfy $L_g^\tau L_h^\tau = \tau(g, h)^{-1} L_{gh}^\tau$. Apply this to $\tau = \bar{\omega}^{-1}$, where $\bar{\omega} = \omega/|\omega|$ is the pointwise phase of ω ; for finite G it is cohomologous to ω . The result is a unitary projective representation with factor system $\bar{\omega}$, which is cohomologous to ω (`TNLean.Algebra.ScalarCocycle.regularProjectiveForClass`, `TNLean.Algebra.ScalarCocycle.regularProjectiveForClass_mem_unitaryGroup`). Feeding it to the construction gives `MPSTensor.exists_unitary_sptFixedPoint_of_isCocycle`: from ω and an arbitrary character ϕ , the fixed point of bond dimension $|G|$ is symmetric up to ϕ under a unitary physical action, `MPSTensor.sptFixedPointAction_mem_unitaryGroup`. Unit modulus of $\phi(g)$ is derived from finite order. Every virtual projective representation realizing this symmetry with the same character has a factor system cohomologous to ω .

The factor system of the supplied representation is $\bar{\omega}$, not ω itself. The source’s claim concerns the class of ω , so the class-level statement is the faithful one. The results that take a representation ρ as input remain available for arbitrary groups. The former trivial-character restriction is now resolved [con26b]; the former on-site restriction is also resolved by the mixed-symmetry construction [con26c]. For time reversal and reflection, a separate semilinear regular representation realizes each prescribed circle-valued twisted factor system exactly, and `MPSTensor.exists_mixed_spt_fixedPoint` supplies the corresponding fixed point. This extension does not change the class-level statement for the general nonzero complex cocycles considered above.

References

- [con26a] The TNLean contributors. SPT fixed points: The printed tensor is not normal. Paper-gap note `rmp_spt_fixed_point_tensor`, TNLean, 2026. https://sirui-lu.com/TNLean/paper-gaps/rmp_spt_fixed_point_tensor.pdf.
- [con26b] The TNLean contributors. SPT fixed points with a nontrivial character. Paper-gap note `rmp_spt_fixed_point_trivial_character`, TNLean, 2026. https://sirui-lu.com/TNLean/paper-gaps/rmp_spt_fixed_point_trivial_character.pdf.
- [con26c] The TNLean contributors. SPT fixed points with time reversal and reflection. Paper-gap note `rmp_spt_fixed_point_onsite_scope`, TNLean, 2026. https://sirui-lu.com/TNLean/paper-gaps/rmp_spt_fixed_point_onsite_scope.pdf.

- [CPGSV21] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product states and projected entangled pair states: Concepts, symmetries, theorems. *Reviews of Modern Physics*, 93(4):045003, 2021.