

The RVB Tensor: the Singlet Matrix on the Vacuum

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Verdict: local-correction (resolved).

1 What the source prints

Appendix A of [CPGSV21], under “Two dimensions: PEPS”, writes the nearest-neighbour resonating valence bond state as a PEPS of bond dimension three. Each site carries the projector

$$P = |0\rangle[(0222| + (2022| + \dots] + |1\rangle[(1222| + (2122| + \dots], \quad (1)$$

whose nonzero entries have exactly one virtual index in a spin state 0, 1 and the other three in the vacuum state 2. The projector is combined with $\pm Y$ on the links, and on the square lattice with a translation-invariant orientation of the singlets the tensor is $A = P(\mathbb{K} \otimes \mathbb{K} \otimes Y \otimes Y)$, where $Y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is the 2×2 singlet matrix introduced for the AKLT state.¹

2 The degenerate reading

The bond has dimension three, while Y is printed as a 2×2 matrix. If Y is extended to the bond by zero on the vacuum, the down and left legs of A can never carry the vacuum index: every column of $\mathbb{K} \otimes \mathbb{K} \otimes Y \otimes Y$ with a vacuum down or left index vanishes, and every other column has at least two spin indices, which (1) annihilates. The tensor A is then zero. The source plainly does not intend this reading, since it describes the state as the superposition of all nearest-neighbour singlet coverings.

3 The correction

The singlet matrix acts on the bond as $Y \oplus 1$: it is Y on the spin states 0, 1 and the identity on the vacuum 2, so the state on each bond is $|01\rangle - |10\rangle + |22\rangle$.

¹Source traceability: `Papers/2011.12127/TN-Review-main.tex`, lines 2434 and 2440–2448.

An edge with vacuum at both ends carries no singlet, and an edge with spin labels at both ends carries a singlet. With this reading, on a torus of width and height at least three, the coefficient of a configuration σ is the sum over nearest-neighbour dimer coverings c of the product of the singlet coefficients $Y_{\sigma_{v+e}, \sigma_v}$ over the edges $(v, v+e)$ of c , with e the unit step to the right or up.²

4 Elimination plan

None needed beyond this record: the extension by the identity on the vacuum is part of the definition of the tensor. Any other nonzero entry c on the vacuum gives the same state up to normalization, since each edge not in a covering carries the vacuum on exactly one down or left leg and every covering of a torus of N sites leaves $3N/2$ edges uncovered, so each covering acquires the same factor $c^{3N/2}$.

References

- [CPGSV21] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product states and projected entangled pair states: Concepts, symmetries, theorems. *Reviews of Modern Physics*, 93(4):045003, 2021.

²Results: `TNLean.PEPS.rvbSingletY`, `TNLean.PEPS.stateCoeff_rvbPEPS`.