

The CZX Tensor: Orientation of the Bond Labels

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Verdict: false-source (open).

1 What the source prints

Section “MPS and PEPS” of [CPGSV21] contracts a PEPS by placing on every edge the maximally entangled pair $\sum_n |n\rangle \otimes |n\rangle$, so the two ends of a bond carry the same label.¹ In Appendix A, under “Two dimensions: PEPS”, the virtual indices of a site tensor are ordered top, right, down and left.² The CZX state of [CLW11] is the product over plaquettes of the four-qubit states $|0000\rangle + |1111\rangle$, and the review writes it, after blocking the four qubits of a site, as the PEPS with tensor

$$\sum_{i,j,k,l=0}^1 |ijkl\rangle ((i,j), (j,k), (k,l), (l,i)), \quad (1)$$

where i, j, k, l are the qubits at the top-left, top-right, bottom-right and bottom-left corners of the site.³

2 The literal contraction

Write (i_v, j_v, k_v, l_v) for the qubits of the site v , and $v + e_1, v + e_2$ for its right and upper neighbours. Contracting (1) with equal labels at the two ends of every bond, the right leg (j_v, k_v) of v meets the left leg (l_{v+e_1}, i_{v+e_1}) of $v + e_1$, and the top leg (i_v, j_v) of v meets the down leg (k_{v+e_2}, l_{v+e_2}) of $v + e_2$. The nonzero coefficients are those with

$$j_v = l_{v+e_1} = l_{v+e_2}, \quad k_v = i_{v+e_1}, \quad i_v = k_{v+e_2} \quad \text{for every site } v. \quad (2)$$

The conditions (2) join the top-right qubit of v to bottom-left qubits of two different sites, and they chain the qubits along the diagonals $v \mapsto v + e_1 - e_2$

¹Source traceability: `Papers/2011.12127/TN-Review-main.tex`, lines 271–276.

²Source traceability: `Papers/2011.12127/TN-Review-main.tex`, line 2415.

³Source traceability: `Papers/2011.12127/TN-Review-main.tex`, lines 2502–2514; `Refereces/1106.4752/source/dDSPTmodel.tex`, lines 317–321.

and $v \mapsto v + e_1 + e_2$. The state is a product of GHZ states on diagonal loops that wind around the torus, not the product of plaquette states. For instance, on a torus of 3×3 sites let σ set the four qubits around one plaquette to 1 and every other qubit to 0. This configuration has coefficient 1 in the product of plaquette states, but the top-right qubit $j_v = 1$ of the site v at the lower left of that plaquette differs from the bottom-left qubit $l_{v+e_2} = 0$ of its upper neighbour, so σ violates (2) and has coefficient 0 in the literal contraction. The printed claim thus fails on nondegenerate data, and the note is filed as a false source claim. The corrected statement below keeps the printed tensor and changes only how its labels are joined across a bond.

3 The correction

Two neighbouring sites list the two qubits of their common side in opposite orders, clockwise around each site. The plaquette state is obtained when a bond identifies the pair (a, b) at one end with (b, a) at the other, so that each bond joins the two qubits of one plaquette to the two qubits of the same plaquette across the common side, as in the figure of the blocked state in [CPGSV21]. Equivalently, the tensor (1) is used with its down and left labels reversed. With this reading the contraction gives

$$c(\sigma) = \prod_v \delta[j_v = i_{v+e_1} = k_{v+e_2} = l_{v+e_1+e_2}], \quad (3)$$

where $c(\sigma)$ is the coefficient of the configuration σ . Formula (3) is the product over plaquettes of the unnormalized states $|0000\rangle + |1111\rangle$, on every torus of width and height at least three.⁴

4 Elimination plan

None needed beyond this record: the printed tensor is kept, and the reversal of the down and left labels is part of the definition of the PEPS. A reading of the source with the reversed identification on every bond would state the same theorem.

References

- [CLW11] Xie Chen, Zheng-Xin Liu, and Xiao-Gang Wen. 2D symmetry protected topological orders and their protected gapless edge excitations. *Phys. Rev. B*, 84:235141, 2011.
- [CPGSV21] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product states and projected entangled pair

⁴Results: `TNLean.PEPS.czxPEPS`, `TNLean.PEPS.stateCoeff_czxPEPS`.

states: Concepts, symmetries, theorems. *Reviews of Modern Physics*, 93(4):045003, 2021.