

Translation-Invariant MPS and the Trivial Phase under Circuits with Measurements: One Direction

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Verdict: scope-restriction (open).

1 What the source states

The paragraph “Phases of matter” of [PSC21] compares sequences of states $\Psi = \{|\psi_M\rangle\}_{M \geq M_0}$ and $\Phi = \{|\varphi_M\rangle\}_{M \geq M_0}$ on chains of increasing size. One writes $\Psi \mapsto \Phi$ if there are $k \in \mathbb{N}$ and states σ_M such that $|\psi_M\rangle$ is mapped to σ_M by a composition of k channels, each a circuit of depth $f(M)$ assisted by measurements and classical communication, and $\|\sigma_M - |\varphi_M\rangle\langle\varphi_M|\|_1 \rightarrow 0$, with f polylogarithmic in M . The sequences are equivalent if $\Psi \mapsto \Phi$ and $\Phi \mapsto \Psi$. The theorem labelled `MPS_classification` reads: “In 1D, all translational invariant MPS with fixed bond dimension are in the same phase as the trivial state.”

2 What is formalized

The formal statement proves the direction from the trivial sequence to the MPS sequence.¹ Unit vectors $|\psi_N\rangle$ are prepared deterministically from a product state by a circuit with measurements followed by a circuit, of total depth at most $C \log N$, and $\| |\psi_N\rangle\langle\psi_N| - |\varphi_N\rangle\langle\varphi_N| \|_1 \rightarrow 0$, where $|\varphi_N\rangle$ is the normalized periodic state. This is $\Psi \mapsto \Phi$ with $k = 2$ and σ_M pure. The restrictions are these.

1. *One direction.* The converse direction, from the MPS sequence to the trivial one, needs channels acting on arbitrary input states, for example exchanging every site with a fresh ancilla in $|0\rangle$ and discarding the ancillas. The measurement model of the formalization prepares states from product states only, and this direction is not stated.

¹Formal declaration: `MPSPreparation.isAsymptoticallyPreparedWithMeasurementsInDepth_normalizedMPVState` in `TNLean/MPS/Preparation/QCccClassification`.

2. *Canonical form.* The tensor is taken in the canonical form $A^i = \bigoplus_j \text{diag}(\mu_{j,1}, \dots, \mu_{j,m_j}) \otimes A_j^i$ of [MSWC24], with every block normal in the gauge $\sum_i A_j^{i\dagger} A_j^i = 1$, with a positive definite fixed point of trace one, and with the mixed transfer maps of distinct blocks of spectral radius below one. The source assumes the canonical form of normal tensors without loss of generality and writes its proof for one normal block; the formal statement does not derive the canonical form of an arbitrary tensor, nor the spectral condition on mixed transfer maps of blocks that are not related by a gauge and a phase. The same reading is recorded in [con26a].
3. *Nonvanishing states.* The periodic states are assumed nonzero for all large N , as a sequence of states presupposes; compare [con26b].

3 Elimination plan

Extend the measurement model to channels acting on an arbitrary input state of the chain together with ancillas that are traced out, prove the direction from the MPS sequence to the trivial one, and derive the canonical-form hypotheses from the canonical form of an arbitrary translation-invariant tensor.

References

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- [MSWC24] Daniel Malz, Georgios Styliaris, Zhi-Yuan Wei, and J. Ignacio Cirac. Preparation of matrix product states with log-depth quantum circuits. *Phys. Rev. Lett.*, 132:040404, 2024.
- [PSC21] Lorenzo Piroli, Georgios Styliaris, and J. Ignacio Cirac. Quantum circuits assisted by local operations and classical communication: Transformations and phases of matter. *Phys. Rev. Lett.*, 127:220503, 2021.