

Asymptotic Local Channel Conversion without Classical Feedforward

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Verdict: scope-restriction (open).

1 The source relation

In the paragraph “Phases of matter” of [PSC21], a sequence $\{|\psi_N\rangle\}$ is convertible into $\{|\varphi_N\rangle\}$ if there is a fixed number k of compositions of channels assisted by measurements and classical communication, each of depth $f(N)$, whose outputs σ_N satisfy

$$\|\sigma_N - |\varphi_N\rangle\langle\varphi_N|\|_1 \longrightarrow 0. \quad (1)$$

Here f is polylogarithmic. Mutual convertibility defines the source’s phase equivalence. The fixed number of compositions and the polylogarithmic depth bound are allowed to depend on the pair of sequences, but not on N .

2 The enlarged-site local-channel model

The available local protocols alternate onsite channels with layers of channels on disjoint nearest-neighbor pairs. They act on arbitrary input operators and allow changes of local dimension. A layer of two-site channels on the resulting enlarged sites costs one depth unit, regardless of that dimension. Intermediate dimensions may depend on the system size without a uniform bound. The model has no classical feedforward between separated sites. For this class, approximate conversion means $\|\Psi(\rho) - \sigma\|_1 \leq \varepsilon$, with protocol depth at most T .¹ For Hermitian input operators, composition adds the depths and errors:

$$\|\Psi_2(\Psi_1(\rho)) - \tau\|_1 \leq \|\Psi_2(\Psi_1(\rho) - \sigma)\|_1 + \|\Psi_2(\sigma) - \tau\|_1 \quad (2)$$

$$\leq \|\Psi_1(\rho) - \sigma\|_1 + \|\Psi_2(\sigma) - \tau\|_1 \leq \varepsilon_1 + \varepsilon_2. \quad (3)$$

The middle inequality is trace-norm contractivity for trace-preserving positive maps on Hermitian operators.

¹Formal declaration: `QuantumCircuit.IsApproxLocalChannelConversion` in `TNLean/Circuit/Channel/ApproximateConversion`.

For families, the error tends to zero and the depth is eventually bounded by $C(1 + \log(N + 1))^p$, for a nonnegative constant C and integer $p \geq 0$. Mutual conversion is an equivalence relation on eventually Hermitian families, and in particular on families of density matrices.² The shift in the logarithm only affects the bound at small system sizes. Finite compositions are already included: their depths add, and a finite sum of polylogarithmic bounds is polylogarithmic. No bound on the dimension of onsite ancillas is imposed.

This gate convention differs from the source. In the Supplement’s proof of Proposition 2 (p. 8), the intersite gates act only on the physical qudits; the entanglement bound across a cut depends on their fixed dimension d , not on the number of onsite ancillas. Allowing one intersite layer to act on an arbitrarily enlarged local space does not by itself give a uniform-depth simulation in that convention. No such simulation or inclusion between the two asymptotic conversion relations is proved here.

3 Blocking sites

In the proof of its classification theorem for matrix product states, the source groups q neighbouring sites of a chain of N sites into one site of dimension d^q , giving a chain of N/q sites [PSC21, Supplemental Material; `main.tex` of arXiv:2103.13367v3, line 287]. An operation on the blocked chain acts on q adjacent qudits of the original chain, and each one- or two-site gate on the blocked chain is realized by fewer than $2q^2$ sequential two-site gates on the original chain [PSC21, `main.tex`, line 289].

Blocking itself is a relabelling of the computational basis. It is a unitary channel and is formalized as a step of depth zero. A blocked conversion first blocks k sites and then runs a local channel protocol of depth at most T on the chain of blocks.³ The blocking factor k is part of the relation: blocking the whole chain into one site and applying a replacement channel there would convert every density matrix into every other one at depth zero. As in the source, the blocking factor is positive. Locality is transported by rescaling: the dual of such a conversion maps an operator acting on a set X of blocks to an operator acting within distance kT of the sites of X in the original chain.

As in the enlarged-site model above, one layer of two-site channels on the blocked chain costs one depth unit. The source’s compilation of a blocked gate into fewer than $2q^2$ nearest-neighbour gates on the original chain is not formalized, so no bound on the original-chain depth of a blocked conversion is asserted.

²Formal declarations: `QuantumCircuit.IsAsymptoticLocalChannelConversion` and `QuantumCircuit.IsLocalChannelPhaseEquivalent.equivalence` in `TNLean/Circuit/Channel/AsymptoticConversion`.

³Formal declaration: `QuantumCircuit.IsBlockedLocalChannelConversion` in `TNLean/Circuit/Channel/SiteBlocking`.

4 Remaining scope

The equivalence proved here belongs to the enlarged-site local-channel model. It is not identified with the source’s phase equivalence or asserted to be a subclass at the same asymptotic depth. The classification asserting that every translation-invariant matrix product state belongs to the trivial source phase is not obtained. Finite adaptive channels on arbitrary input operators and their additive-depth composition are now available separately, as described in [con26a]. They are not included in the non-feedforward asymptotic relation formalized here.

The remaining source comparison needs a decomposition into a fixed number of QCC blocks, with that number independent of system size. Each block must satisfy the source’s internal measurement/control order and one-measurement-per-site rule. The finite adaptive tree does not record this decomposition: counting its individual instrument steps would not express the source condition. The comparison must also distinguish fixed-dimensional physical qudits from onsite ancillas and control the intersite gate cost uniformly in the chain length. A proved simulation bound or explicit restrictions on intermediate dimensions could supply this resource comparison. Once those source-model depth bounds are established, trace-norm contractivity supplies the same error estimate. The separate restrictions of the matrix-product-state classification are recorded in [con26b].

References

- [con26a] The TNLean contributors. Finite adaptive local channels and source protocol blocks. Paper-gap note psc21_adaptive_channel_round_scope, TNLean, 2026. https://sirui-lu.com/TNLean/paper-gaps/psc21_adaptive_channel_round_scope.pdf.
- [con26b] The TNLean contributors. Translation-invariant MPS and the trivial phase under circuits with measurements: One direction. Paper-gap note psc21_mps_classification_scope, TNLean, 2026. https://sirui-lu.com/TNLean/paper-gaps/psc21_mps_classification_scope.pdf.
- [PSC21] Lorenzo Piroli, Georgios Styliaris, and J. Ignacio Cirac. Quantum circuits assisted by local operations and classical communication: Transformations and phases of matter. *Phys. Rev. Lett.*, 127:220503, 2021.