

Finite Adaptive Local Channels and Source Protocol Blocks

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Verdict: scope-restriction (open).

1 Source operations

Pirolì, Styliaris and Cirac [PSC21], paragraph “State transformations with QC and LOCC”, define one circuit followed by sequential local measurements and local unitaries conditioned on earlier outcomes. The same paragraph mentions the possible extension to several rounds of local operations and classical communication. The later paragraph “Phases of matter” allows a number of composed channels that is fixed independently of the system size.

2 Finite adaptive channels

An onsite instrument has local Kraus operators $K_{i,j}$. At each site they satisfy $\sum_j K_{i,j}^\dagger K_{i,j} = I$. For an outcome tuple J , let $K_J = \bigotimes_i K_{i,J(i)}$. If Ψ_J is the channel selected by that tuple, averaging the outcomes gives

$$\rho \mapsto \sum_J \Psi_J(K_J \rho K_J^\dagger). \quad (1)$$

These are unnormalized operations. Their traces already contain the outcome probabilities, so no division by probabilities is needed, even on branches whose probability is zero. Complete positivity follows from the Kraus operators of the continuations, composed with K_J . Trace preservation follows from $\sum_J K_J^\dagger K_J = I$ and trace preservation of each Ψ_J .¹

A finite adaptive protocol is built from onsite channels, layers of nearest-neighbor channels, and onsite instruments followed by outcome-dependent continuations. Its depth bounds the number of nearest-neighbor layers on every branch; onsite operations and classical communication cost zero depth. Repeated instrument steps allow later measurements and gates to depend on the

¹Formal declarations: `QuantumCircuit.OnsiteChannel.feedforwardMap` and `QuantumCircuit.OnsiteChannel.feedforwardMap_isKrausCPTP` in `TNLean/Circuit/Channel/Feedforward`.

complete history. Such protocols are channels on arbitrary input operators. Composing protocols of depth at most T_1 and T_2 has depth at most $T_1 + T_2$.²

The existing fixed-basis measurement round has Kraus operators $V_m P_m U$, where U is its circuit, the orthogonal projections P_m sum to I , and V_m is the product of conditional local corrections. Its Kraus operators satisfy

$$\sum_m (V_m P_m U)^\dagger (V_m P_m U) = I. \quad (2)$$

Thus its average map is a channel on arbitrary inputs.³ The round is also a finite adaptive protocol of depth equal to its circuit length: an onsite instrument measures the selected sites, then the outcome selects the onsite correction.⁴

3 Physical gate-cost boundary

The adaptive predicate inherits the enlarged-site convention of the local channel model. Onsite channels may change the local dimension without a uniform bound, and later gates on a pair of enlarged sites still cost one layer. The source distinguishes fixed-dimensional physical qudits from onsite ancillas. In particular, the Supplement’s proof of Proposition 2 (p. 8) bounds the entanglement increase using the physical dimension, independently of the number of onsite ancillas.

Consequently, the formal quantum-depth bound is not yet a bound in the source’s resource model. A comparison needs an explicit simulation by physical-qudit gates, with a depth overhead controlled uniformly in the chain length. A possible restricted target bounds every intermediate onsite dimension by one constant before quantifying over chain lengths, then derives an explicit simulation overhead depending on that constant and the physical dimension. No such simulation is established here.

4 Asymptotic boundary

The source’s asymptotic relation fixes the number of composed *QCC* protocols independently of system size. This is not a bound on individual local measurement/correction steps: one source protocol can already contain sequential steps at many sites. Each block must also obey its internal measurement/control order and one-measurement-per-site rule.

The finite adaptive tree does not record a decomposition into a fixed number of such source blocks. Its bound counts quantum layers, while instrument steps

²Formal declarations: `QuantumCircuit.IsAdaptiveChannelProtocol`, `QuantumCircuit.IsAdaptiveChannelProtocol.isKrausCTP`, and `QuantumCircuit.IsAdaptiveChannelProtocol.comp` in `TNLean/Circuit/Measurement/AdaptiveConversion`.

³Formal declarations: `QuantumCircuit.MeasurementRound.map` and `QuantumCircuit.MeasurementRound.map_isKrausCTP` in `TNLean/Circuit/Measurement/Channel`.

⁴Formal declaration: `QuantumCircuit.MeasurementRound.adaptive` in `TNLean/Circuit/Measurement/AdaptiveRound`.

are free and may be interleaved with later quantum operations. A comparison therefore needs an actual source-block decomposition, with its block bound quantified before chain length, in addition to the physical gate-cost comparison. Merely counting instrument nodes would not express the source condition. The finite-tree channel results, the source-block asymptotic relation, and the MPS classification are separate statements.⁵

References

- [PSC21] Lorenzo Piroli, Georgios Styliaris, and J. Ignacio Cirac. Quantum circuits assisted by local operations and classical communication: Transformations and phases of matter. *Phys. Rev. Lett.*, 127:220503, 2021.

⁵Finite adaptive conversions are tracked by GitHub issue #8382; the asymptotic relation and classification are tracked under GitHub issue #8622.