

# Party-Network Bounds in the Dyadic Routing Lemma

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**Verdict:** scope-restriction (open).

## 1 The source lemma

Lemma 8.2 of the polynomial PEPS approximation manuscript of September 24, 2026 [Ope26] (`07-assembly.tex`, lines 122–127) states, with no hypotheses, that the links of the party network can be routed on the padded grid  $\{0, \dots, N-1\}^2$  with constant edge congestion, and that after reflection into the genuine  $L \times L$  grid the congestion is still constant and the physical output coordinates are unchanged. The lemma concerns one specific network: the party network built by the distribution protocol of Proposition 7.1 (`06-geometry.tex`, lines 12–53), with its parties attached to the anchors of the dyadic blocks (`07-assembly.tex`, lines 103–114). The proof (lines 128–162) uses four properties of that network:

1. every anchor lies on the padded grid (lines 113–114);
2. at most a bounded number  $B$  of parties is attached to any dyadic block (lines 113–114, from item 1 of Proposition 7.1);
3. every party has at most a bounded number  $\Delta$  of incident links (lines 140–141, from item 2 of Proposition 7.1);
4. the anchors of the two endpoints of every link are at distance at most  $c$  times either endpoint scale, for a fixed  $c$  (lines 130–132, from item 3 of Proposition 7.1).

Proposition 7.1 also gives that the two endpoint scales of every link are equal or adjacent; the congestion bound does not use this.

## 2 The formal statement

Proposition 7.1 is not yet formalized, so no formal party network of the protocol exists to which the lemma could refer. The formal statements are therefore made for an arbitrary party network with parties attached to dyadic blocks, and the

four properties above are hypotheses: the anchors lie on the padded grid, at most  $B$  parties per block, at most  $\Delta$  links per party, and link separation at most  $c$  times either endpoint scale. Under these hypotheses the padded congestion is at most  $2(2c + 1)B\Delta$ , the folded congestion is at most  $8(2c + 1)B\Delta$ , and the routed network is exactly a PEPS on the open  $L \times L$  square lattice with bond dimension at most  $\max(D, 1)^{8(2c+1)B\Delta}$  when every link has dimension at most  $D$ .<sup>1</sup>

The constants depend only on  $B$ ,  $\Delta$  and  $c$ , as in the source. The formal statements are a conditional form of Lemma 8.2, valid for every party network with the four properties; the lemma for the network of the source is the special case in which the four properties hold.

### 3 Elimination

Formalize Proposition 7.1 and the attachment of its parties to the anchors of the dyadic blocks of the padded hierarchy. The two sources of the four properties are then discharged separately. Property 1, that every anchor lies on the padded grid, follows from the attachment itself (`07-assembly.tex`, lines 113–114) and is not a clause of Proposition 7.1. Properties 2–4, the block, degree and separation bounds, follow from items 1–3 of Proposition 7.1, with constants uniform in the Hamiltonian and in  $L$ . Applying the conditional statements to that network then gives Lemma 8.2 as stated.

### References

- [Ope26] OpenAI. Polynomial PEPS approximation of gapped square-grid ground states, 2026. September 24, 2026 manuscript.

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<sup>1</sup>The padded bounds are `TNLean.PEPS.Approximation.DyadicPlacement.load_horizontal_le` and `TNLean.PEPS.Approximation.DyadicPlacement.load_vertical_le`; the folded bound is `TNLean.PEPS.Approximation.foldedRouting_card_traversal_le`; the PEPS assembly is `TNLean.PEPS.Approximation.exists_squarePEPS_of_dyadicRouting`.