

Nonempty parties in distributed density compression

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Verdict: local-correction (resolved).

Theorem 5.2 of the September 24, 2026 polynomial-PEPS manuscript concerns compression to an operator tensor network with one node per party [Ope26, Theorem 5.2]. The statement does not explicitly require the party set to be nonempty. The formalization adopts this requirement once, in the definition of a *distributed construction*. It is the intended convention needed by the scalar-absorption step of the proof, rather than an assertion that the printed statement includes this qualifier.

The scalar boundary case. The preliminaries state that empty tensor products have dimension one (01-preliminaries.tex, lines 5–7) [Ope26, preliminaries]. Accordingly, a circuit on no parties can still act on $\mathbb{C}$ by a scalar contraction c . Starting from the empty product input 1, its physical density is the scalar $|c|^2$. An operator tensor network with no nodes or links contracts to the empty product 1. These are different when, for example, $c = 1/2$; their absolute difference is $3/4$. Thus the literal zero-party reading does not imply approximation to arbitrarily small error by a network having exactly one node per party. No zero-party compression theorem is asserted.

The adopted convention. A distributed construction has a finite nonempty set of parties. The original memories are finite-dimensional, the initial vector is a product of normalized party vectors, and the circuit retains the private output registers until the final trace, exactly as in 04-compression.tex, lines 17–30 and 137–151 [Ope26, Theorem 5.2]. No additional bound is imposed on private dimensions. The network construction chooses local tensors at existing parties and absorbs scalar coefficients and averaging factors into them (lines 565–578) [Ope26, proof of Theorem 5.2]. In particular, the scalar contributed by a gate with no participating parties can be absorbed at any existing party, without introducing a node or a virtual link.

The general circuit definition remains unrestricted: gates with no participating parties, their chronological occurrences, and their scalar contributions are all retained. Only the definition of a distributed construction carries the nonempty-party convention; it is not repeated as a hypothesis of each subsequent theorem. The spatial families used for square grids of side length $L \geq 2$ already have a nonempty party set, so this convention does not change those applications.

Source. The cited source is the manuscript at `openai/math@adc7f1241b42`, *Polynomial PEPS approximation of gapped square-grid ground states*, the preliminaries and the section on distributed density compression [Ope26].

References

- [Ope26] OpenAI. Polynomial PEPS approximation of gapped square-grid ground states, 2026. September 24, 2026 manuscript.