

Policy for Paper-Gap Notes

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1 Purpose

A paper-gap note records a mathematical discrepancy between a cited source and the statement, proof, or route used in the formal development. The source itself should remain intact. If a proof seems to require a hypothesis not present in the article, a different scalar, a different normalization, or a replacement theorem from the formal library, the change should be explained in a separate note under `docs/paper-gaps/`.

The note is a mathematical document first. It should be readable by a mathematician or mathematical physicist who has not read the corresponding issue or pull request. Traceability data such as formal declaration names, source-file paths, GitHub issues, and line ranges should support the exposition, usually in footnotes, rather than govern the exposition.

2 When a note is needed

A note is needed when the formal statement is not literally the cited statement. The common case is an implication in the paper,

$$\mathcal{H} \implies C, \quad (\text{source})$$

while the formal proof establishes only

$$\mathcal{H} \wedge \mathcal{K} \implies C. \quad (\text{formal})$$

The condition $\mathcal{K}$ must then be accounted for. Either it is derived from $\mathcal{H}$, it is a harmless local correction, or the formal theorem is a restricted theorem and should be marked as such.

A note is also needed when the proof is mathematically different even though the final conclusion is the same. For example, a source proof might identify a coefficient by a finite power-sum identity,

$$\sum_{q=1}^{r_a} \mu_q^N = \sum_{q=1}^{r_b} \nu_q^N \quad (1 \leq N \leq m), \quad (1)$$

whereas the formal route might use convergence of a sequence. These are different mechanisms, and the note should say which hypotheses each mechanism uses.

3 Form of the note

The note should introduce the relevant notation before using it. It should state the cited assertion in the special case under discussion, then display the formal or corrected statement, and finally compare the hypotheses and conclusions. Long quotations are usually unnecessary; a precise citation and the mathematical formula are better.

Displayed equations should carry the calculation. Use `align` for displayed formulae in these notes, especially for multi-line arguments:

$$V^{(N)}(A) = c_N V^{(N)}(B), \quad (2)$$

$$\langle V^{(N)}(B_k), V^{(N)}(A_j) \rangle \xrightarrow{N \rightarrow \infty} 0, \quad (3)$$

$$\dim \text{span} \left(\{V^{(N)}(A_j)\}_j \cup \{V^{(N)}(B_k)\} \right) = r_A + 1. \quad (4)$$

The contradiction should then be explained in prose, for instance by saying which nonzero coefficient is forced to vanish. Source tags belong on the right-hand side of the displayed line, using `\tag` when a tag is needed. Short definitions may remain inline when the sentence is clearer that way.

Formal identifiers should be written with `\leanid` and should usually appear in footnotes. For example, the prose may say that the formal theorem is the same-dimensional gauge-phase component of a source lemma, and the footnote may identify the declaration.¹ Do not put formal identifiers in displayed equations unless the mathematical object itself is a formal declaration.

4 Shared commands

All notes in this directory should load `docs/paper-gaps/command.tex`. Shared notation belongs there. In particular,

$$\text{tr}(X), \quad \text{someDeclaration} \quad (5)$$

should be produced by the shared commands `\tr` and `\leanid` rather than by local redefinitions in each note. This keeps the documents visually consistent and avoids contradictory local conventions.

¹Formal declaration: `MPSTensor.gaugePhaseEquiv_of_overlap_norm_tendsto_one_of_irreducible_TP` in `TNLean/MPS/FundamentalTheorem/Proportional`.

5 Naming

A note’s file name has the form `<key>_<topic>.tex`. The key names the source the note examines: the authors’ initials followed by a two-digit year (`cpsv16`, `pgvwc07`, `spwc10`), or an established short name for the source—a book or review (`wolf`, `rmp`) or a paper known by its subject (`mpu` for matrix product unitaries, `peps` for normal PEPS). A key may also name a small fixed collection of companion papers examined together (`cpsv17`). A note that audits the formal development itself, with no single external source, takes the key `tnlean`. The registry of keys, each with the source it names, is maintained in the repository configuration (`texra-blueprint.toml`), and the index generator rejects an unregistered key.

The topic part states the mathematical subject, not the history of the note. A file name is permanent once other documents cite it; version suffixes are not used, since a note is revised in place (see Revision below) and its history is preserved by the repository.

Blueprint prose cites a note like any other source, with `\cite{gap:<slug>}`; the bibliography entry carries the published URL of the note, whether the note lives in this repository or in another one. Lean docstrings and comments use the repository path form `docs/paper-gaps/<slug>.tex`.

6 Classification

The conclusion of a note should give a verdict. The usual classifications are: a notational clarification, a local correction, a scope restriction, an unfaithful theorem, a source claim that is false as printed, or an open gap. The classification should follow the mathematics, not the amount of formal work required. Each note declares its classification and its status machine-readably with `\gapnote{<kind>}{<status>}` directly after `\maketitle`: the kind in the kebab-cased vocabulary above (`clarification`, `local-correction`, `scope-restriction`, `unfaithful`, `false-source`, `open-gap`) and the status one of `open`, `wip`, `resolved`, or `historical`; `wip` marks a gap whose elimination is actively underway and still counts as live. Severity is derived from the kind, so it is never stated separately. A resolved note keeps its classification and changes only its status; the published index lists live notes first and dims the resolved ones.

A local correction records a small adjustment whose corrected statement is still available in the cited argument. A scope restriction records a theorem proved only in a special case of the source theorem. An unfaithful theorem is more serious: it relies on a hypothesis or intermediate result that the cited source does not provide. Such a theorem must carry an in-source marker pointing to the paper-gap note. A source claim is false as printed when a counterexample satisfies the stated source hypotheses and contradicts its conclusion. This classification concerns the cited assertion itself, rather than a mismatch introduced by the formal statement or proof.

7 Revision

A note may begin provisionally, but revisions should keep it as a coherent account of the present mathematical understanding. It should not become a chronological log. If later work proves that $\mathcal{K}$ follows from $\mathcal{H}$, the note should be rewritten to show that derivation. If later work confirms that the extra hypothesis is unavoidable, the note should state the restricted theorem explicitly.

References