

Scope of the Formal Block-Separation Lemmas Relative to PGVWC07 Uniqueness

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Verdict: scope-restriction (open).

1 Source theorem

The translation-invariant canonical-form uniqueness theorem of Pérez-García, Verstraete, Wolf, and Cirac starts with two TI canonical D -MPS representations of the same finite-ring state. We write their matrices as B_i and C_i , following the source theorem [PGVWC07, Theorem `thm-uniq`, lines 1002–1015]. For a TI representation A_i , PGVWC07 defines

$$\Gamma_L(X) = \sum_{i_1, \dots, i_L} \text{tr}[X A_{i_1} \cdots A_{i_L}] |i_1 \cdots i_L\rangle.$$

The finite injectivity hypothesis called C1 in the source is the existence of a finite length L_0 for which Γ_{L_0} is injective [PGVWC07, lines 888–908]. The theorem assumes:

1. this finite injectivity hypothesis holds;
2. the open-boundary-condition (OBC) canonical representation of the MPS state is unique;
3. the length satisfies $N > 2L_0 + D^4$.

The conclusion is the existence of a unitary matrix U such that

$$B_i = U C_i U^\dagger$$

for every physical index i , with the usual permutation-and-phase interpretation when the Schmidt weights are non-degenerate.

2 Formal boundary

The checked block-separation lemmas in Chapter 9 are not formalizations of PGVWC07 Theorem `thm-uniq`. They are conditional same-structure comparison results. Their hypotheses include blockwise normalization, injective or irreducible block hypotheses, a common block index set and dimension function, and strict ordering of the weight moduli.¹

These assumptions are useful local inputs for later MPS arguments, but they are not the hypothesis set of the PGVWC07 uniqueness theorem. In particular, the source theorem does not assume a same-structure block decomposition of the two representations, nor does it assume the Chapter 9 strict-modulus separation lemma as an input. Its proof uses the OBC canonical representation and the Horn–Johnson matrix lemma route described after [PGVWC07, Theorem `thm-uniq`, lines 1002–1015].

Thus this is a theorem-level gap. PGVWC07 Theorem `thm-uniq` is not currently formalized. The available formal lemmas are conditional same-structure consequences with stronger hypotheses, and therefore must not be cited as formalizations of that source theorem.

3 Required formalization

A source-faithful formalization of PGVWC07 Theorem `thm-uniq` should state the finite-ring injectivity hypothesis above, uniqueness of the OBC canonical representation, and $N > 2L_0 + D^4$, and should conclude a single unitary intertwiner between the two TI canonical D -MPS representations. The current Chapter 9 block-separation lemmas may be used only as local consequences after their stronger hypotheses have been proved in the argument at hand.

4 Tracked consequences

Blueprint statements that point to Chapter 9 block separation must explicitly state the strict weight-modulus hypothesis when their Lean declarations require it. Downstream CPSV16 uses should not cite these statements as PGVWC07 canonical-form uniqueness unless the finite-injectivity, OBC

¹The corresponding formal declarations are `MPSTensor.per_block_sameMPV_of_canonical_form`, `MPSTensor.per_block_sameMPV_of_normal_canonical_form`, `MPSTensor.fundamentalTheorem_canonicalForm`, and `MPSTensor.fundamentalTheorem_canonicalForm_explicit`.

uniqueness, finite-size-bound, and unitary-conclusion theorem have been formalized separately.

References

- [PGVWC07] David Pérez-García, Frank Verstraete, Michael M. Wolf, and J. Ignacio Cirac. Matrix product state representations. *Quantum Information and Computation*, 7(5–6):401–430, 2007.