

Positive Block Dimensions in the PGVWC07 Canonical Form

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Verdict: local-correction (resolved).

1 The printed block form

Theorem 4 of Ref. [PGVWC07] (`Papers/quant-ph_0608197/MPsarchive.tex`, lines 742–763) writes the matrices of a translation-invariant canonical representation as

$$A_i = \bigoplus_j \lambda_j A_i^j, \quad 1 \geq \lambda_j > 0, \quad (1)$$

where every block A^j satisfies $\sum_i A_i^j (A_i^j)^\dagger = \mathbb{1}$, admits a diagonal positive full-rank matrix Λ^j with $\sum_i (A_i^j)^\dagger \Lambda^j A_i^j = \Lambda^j$, and has the identity as the only fixed point of $X \mapsto \sum_i A_i^j X (A_i^j)^\dagger$. The source does not state the size of a block.

2 The convention

A block of size zero contributes nothing to (1) and is not a summand the source intends: its weight λ_j would be a listed weight of no matrix, and its conditions would hold vacuously. The formal predicate for the block form of Theorem 4 therefore requires every listed block to have positive bond dimension. This is a convention of the same kind as the nonzero-coefficient convention for canonical forms, not a faithfulness gap: the predicate admits exactly the block forms the source describes, and the existence theorem for the canonical form produces only blocks of positive dimension.

The convention is used once. In the uniqueness theorem for the canonical form (Theorem 7, lines 1002–1015), condition C1 forces a single block, by the first assertion of the proposition on condition C1 (lines 911–919): a matrix unit supported on an off-diagonal block is annihilated by every word. Choosing that matrix unit requires one bond index in each of two distinct blocks, which positive block dimension supplies.¹

¹The formal declarations are `MPSTensor.PGVWC07CanonicalFormData` and `MPSTensor.PGVWC07CanonicalFormData.not_isNBlkInjective_of_ne`.

References

- [PGVWC07] David Pérez-García, Frank Verstraete, Michael M. Wolf, and J. Ignacio Cirac. Matrix product state representations. *Quantum Information and Computation*, 7(5–6):401–430, 2007.