

Scope of the Formal Site-Independent Construction Relative to PGVWC07

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Verdict: scope-restriction (open).

1 Source theorem

Pérez-García, Verstraete, Wolf, and Cirac prove that every translation-invariant pure state on a finite periodic chain has an MPS representation with site-independent matrices [PGVWC07, Theorem 3]. Their proof first obtains a varying-bond open-boundary representation by successive Schmidt or singular value decompositions. It then pads the rectangular virtual spaces to one square space and averages the resulting chain over cyclic translations. If N is the ring length and D bounds the intermediate bond dimensions, the site-independent tensor has bond dimension ND .

2 Formal boundary

The structure `OBChainTensor` records a supplied varying-bond open-boundary representation. The theorem `OBChainTensor.pgvlc07_site_independent_matrices` proves that, for a positive fixed length N , cyclic averaging of its zero-padding has the same coefficients whenever those coefficients are translation invariant. Its existence form is `OBChainTensor.exists_pgvlc07_site_independent_tensor`.

Thus the formal theorem proves the square-padding and cyclic-averaging part of PGVWC07 Theorem 3. It does not construct an `OBChainTensor` from an arbitrary pure state. In particular, purity is not a hypothesis of the formal theorem because the representation whose existence purity supplies in the source is already an input.

3 Elimination plan

Formalize the finite-chain Schmidt decomposition that expresses an arbitrary pure state as a varying-bond open-boundary MPS with endpoint bond dimen-

sions one. Bound every intermediate bond dimension by a common D , instantiate `OBCCChainTensor`, and prove that translation invariance of the state gives `OBCCChainTensor.IsTranslationInvariantState`. Composing that existence theorem with `OBCCChainTensor.exists_pgvwc07_site_independent_tensor` will yield the source-facing statement for arbitrary translation-invariant pure states. Issue #6086 tracks this missing existence theorem.

4 Verdict

Verdict: scope restriction. The formal declarations prove the fixed-length zero-padding and cyclic-averaging construction conditional on a supplied varying-bond open-boundary representation. The Schmidt-decomposition argument producing that representation from an arbitrary pure state remains open, so the current result is not the full source theorem.

References

- [PGVWC07] David Pérez-García, Frank Verstraete, Michael M. Wolf, and J. Ignacio Cirac. Matrix product state representations. *Quantum Information and Computation*, 7(5–6):401–430, 2007.