

Sequential Generation With an Ancilla: Nonempty Chains and Ancillas

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Verdict: local-correction (resolved).

1 Source range

The theorem “Sequential generation with ancilla” of Ref. [PGVWC07, Section 5, lines 1569–1573] states that “the three sets of multipartite states which can be generated by the above sequential schemes with ancilla are all equal to the set of states with OBC MPS representation with maximal bond dimension D .” The theorem carries no lower bound on the number N of sites or on the ancilla dimension D .

2 The degenerate cases

For $N = 0$ the only coefficient of an open-boundary representation, whose first and last bond dimensions are one, is the empty product 1. The open-boundary states on the empty chain are therefore the vector 1 only, while the probabilistic scheme with $D \geq 1$ generates every scalar $\langle \varphi_F | \varphi_I \rangle$ and the deterministic and transition schemes generate every scalar of modulus one. For $D = 0$ the ancilla space is zero, every generated vector is zero, and no vector has an open-boundary representation with bond dimension at most 0. Neither case is one the authors intend: the section concerns spin chains of $N \geq 1$ sites and an ancilla that carries a state.

3 Convention

The equalities of sets are formalized for chains of $N \geq 1$ sites, and the equality for the probabilistic scheme also for ancillas of dimension $D \geq 1$. The deterministic and transition schemes need no lower bound on D , since their normalization conditions are unsatisfiable when $D = 0$. On the empty chain the two directions of the probabilistic and deterministic equalities are formalized up to a nonzero scalar, which is the exact form of the statement there.

References

- [PGVWC07] D. Pérez-García, F. Verstraete, M. M. Wolf, and J. I. Cirac. Matrix product state representations. *Quantum Information & Computation*, 7(5):401–430, 2007.