

Sequential Generation Without an Ancilla Needs Two Sites

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Verdict: local-correction (resolved).

1 Source range

The theorem “Sequential generation without ancilla” of Ref. [PGVWC07, Section 5, lines 1580–1616] states that the pure states generated by a sequential scheme without an ancilla, either deterministically or probabilistically, are exactly the states with an open-boundary matrix product representation of bond dimension $D \leq d$. The scheme starts from $|0\rangle^{\otimes N}$ and performs “first an operation affecting the sites 1 and 2, then 2 and 3 up to one between $N - 1$ and N ” (lines 1583–1586). The theorem carries no lower bound on N .

2 The one-site chain

The scheme presupposes at least two sites: its first operation acts on sites 1 and 2. Read literally at $N = 1$, the scheme performs no operation and generates only $|0\rangle$, while every vector of $\mathbb{C}^d$ is an open-boundary matrix product state of bond dimension one. The equality of the two classes therefore fails at $N = 1$ for every $d \geq 2$, and it is not the case the authors intend.

3 Convention

The formalization states the scheme and the theorem for chains of length $N = n + 2 \geq 2$, where the scheme is defined as printed. For every such N the theorem is formalized with the source’s hypotheses. The source’s proof of the first direction further separates the last operation $U^{[N-1]}$, whose matrices it obtains by a singular value decomposition; the formal proof treats all operations uniformly and reads the first ket position from the output vector, which gives the same open-boundary representation.

References

- [PGVWC07] D. Pérez-García, F. Verstraete, M. M. Wolf, and J. I. Cirac. Matrix product state representations. *Quantum Information & Computation*, 7(5):401–430, 2007.