Common Identity Coefficients in the Block Ground-Space Intersection

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Verdict: scope-restriction (resolved).

1 The source argument

The proof of Theorem 12 in [PGVWC07, lines 1424–1456] compares two boundary representations of a vector in the intersection

$$\mathbb{C}^d \otimes \left(\bigoplus_j \mathcal{G}_m^{A^j} \right) \cap \left(\bigoplus_j \mathcal{G}_m^{A^j} \right) \otimes \mathbb{C}^d.$$

The common word-span condition gives, for every block j and physical indices a, b,

$$A_b^j C_a^j = D_b^j A_a^j.$$

The source then uses the right-canonical normalization

$$\sum_a A_a^j A_a^{j\dagger} = I.$$

Its displayed definition $E^j = \sum_a C_a^j A_a^j$ omits the adjoint required by the following calculation. With the adjoint-corrected definition

$$E^j = \sum_a C_a^j A_a^{j\dagger},$$

one obtains

$$D_b^j = A_b^j E^j, \qquad A_b^j C_a^j = A_b^j E^j A_a^j.$$

This source-aligned argument is formalized by `MPSTensor.pgvwc07_mem_iSup_groundSpace_of_trace_decomposition`.

2 The common-coefficient variant

The local variant `MPSTensor.pgvwc07_mem_iSup_groundSpace_of_trace_decomposition_of_identity_coefficients` does not assume right-canonical normalization. Instead, it assumes common coefficients c_a, independent of the block j, such that

$$\sum_a c_a A_a^j = I$$

for every block, and defines

$$E^j = \sum_a c_a C_a^j.$$

Summing $A_b^j C_a^j = D_b^j A_a^j$ against c_a gives

$$A_b^j E^j = D_b^j,$$

and hence the same boundary identity

$$A_b^j C_a^j = A_b^j E^j A_a^j.$$

Thus the conclusion agrees with the local step of the source proof, but the hypothesis and construction of E^j are different.

3 Formal boundary

The common-coefficient theorem is a scope restriction: it is a conditional local variant, not the formalization of the right-canonical step printed in Theorem 12. Results that derive the common coefficients from a simultaneous one-letter word-span condition may use this variant. A source-faithful proof should instead use `MPSTensor.pgvwc07_mem_iSup_groundSpace_of_trace_decomposition`, which retains the right-canonical hypothesis and the adjoint-corrected boundary matrix. No mathematical hypothesis should be transferred between these two statements without a separate implication.

References

[PGVWC07] David Pérez-García, Frank Verstraete, Michael M. Wolf, and J. Ignacio Cirac. Matrix product state representations. *Quantum Information and Computation*, 7(5–6):401–430, 2007.