

Compatible Coherent Seeds and Non-Normal State Conversion

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Verdict: scope-restriction (open).

1 The unresolved interpretation

The discussion of [MSWC24] states that MPS in the same phase can be transformed into each other with a logarithmic-depth circuit. It does not specify a non-normal phase predicate or how a selected vector in a degenerate groundspace is transported. Issue 8469 asks for a circuit between normalized periodic vectors, without a compatibility condition on their sector amplitudes. The obstruction below concerns the interpretation that equality of parent groundspaces suffices for this chosen-vector claim. It does not refute every interpretation of the paper’s informal remark.

2 A quantitative obstruction

Consider the tensors

$$A^0 = \text{diag}(1, 1, 0), \quad A^1 = \text{diag}(0, 0, 1), \quad (1)$$

$$B^0 = \text{diag}(1, 0), \quad B^1 = \text{diag}(0, 1). \quad (2)$$

For every positive length their normalized periodic vectors are

$$\psi_N = (2|0^N\rangle + |1^N\rangle)/\sqrt{5}, \quad \phi_N = (|0^N\rangle + |1^N\rangle)/\sqrt{2}. \quad (3)$$

Both have local parent support $\text{span}\{|00\rangle, |11\rangle\}$ and the same ferromagnetic parent groundspace. All copy weights are nonzero. The first tensor and its periodic-vector formula already occur in `TNLean/MPS/Preparation/RepeatedBlockCounterexample`; no claim about its previously corrected polar approximation is needed here.

Let U be a depth- T nearest-neighbor unitary circuit on the ring, and assume $N > 4T + 4$. Choose two sites whose backward light cones are disjoint and whose union leaves at least one site unobserved. The pulled-back Pauli observables $X = U^\dagger Z_i U$ and $Y = U^\dagger Z_j U$ have norm one and are supported in

those respective cones. Their cross terms between $|0^N\rangle$ and $|1^N\rangle$ vanish, as do those of XY , because of the unobserved site. Expectations of XY factor within each product branch. Writing $p = 4/5$, $a_k = \langle k^N | X | k^N \rangle$ and $b_k = \langle k^N | Y | k^N \rangle$, with $a_k, b_k \in [-1, 1]$, gives

$$\text{Cov}_{U\psi_N}(Z_i, Z_j) = p(1-p)(a_0 - a_1)(b_0 - b_1) \leq 16/25. \quad (4)$$

The covariance in ϕ_N equals one. Choose a scalar $|c| = 1$ minimizing $\eta = \|U\psi_N - c\phi_N\|$. Every unitary observable has expectation difference at most 2η , by expanding the difference into two inner products and applying Cauchy-Schwarz. The covariance difference is at most 6η , so $\eta \geq 3/50$. Phase alignment gives

$$\eta^2 = 2(1 - |\langle U\psi_N | \phi_N \rangle|), \quad 1 - |\langle U\psi_N | \phi_N \rangle| \geq 9/5000. \quad (5)$$

Thus overlap error strictly below $9/5000$ requires $N \leq 4T + 4$. Fixing an accuracy below this bound excludes any depth $T \leq c \log(N/\varepsilon)$ for all sufficiently large N . The model is a closed nearest-neighbor unitary circuit on the physical ring; measurements and nonlocal classical feedforward are different resources.

Formal statements. The explicit-vector lower bound is `MPSPreparation.hz_sector_weight_infidelity_lower_bound` in `TNLean/MPS/Preparation/GHZSectorObstruction`. The state formulas and equality of all local parent spaces, hence the canonical parent Hamiltonians, are proved in `TNLean/MPS/Preparation/GHZSectorParent`. The sector coefficient two comes from two identical virtual copies, each with scalar weight one; it is a trace multiplicity independent of N . The existing polar-approximation counterexample is not used to infer a circuit lower bound. The same-phase scope question remains attached to issue 8469 because this obstruction concerns selected vectors with incompatible sector probabilities, rather than every phase definition.

3 The sufficient statement

Suppose normalized vectors a, b are approximated by $U_A \chi_A, U_B \chi_B$ with errors $\varepsilon_A, \varepsilon_B$, where both seeds are normalized and $R\chi_A = \chi_B$. Require U_A, U_B, R to be actual circuits on one common physical ring, of depths T_A, T_B, T_R . Then

$$U = U_B R U_A^\dagger, \quad T_U \leq T_A + T_R + T_B, \quad (6)$$

$$1 - |\langle Ua | b \rangle| \leq 2(\varepsilon_A + \varepsilon_B). \quad (7)$$

The common-seed case has $R = I$, $T_R = 0$. Taking endpoint errors at most $\varepsilon/4$ and uniform depth bounds yields $O(\log(N/\varepsilon))$. The constants must precede both N and ε in the quantifiers. No symmetry constraint is imposed, and no constant-depth or exact target conversion is claimed. Any auxiliary registers must be included with their specified reset vectors; reduced-state equality is insufficient to invert a preparation.

The coherent circuit extracted from the non-normal preparation implements all encoded sector basis vectors simultaneously; its choice does not depend on their amplitudes. Its input includes zero-valued unused sites and its output is the full block-isometry vector. Its depth is at most CL for physical block lengths bounded by L . An arbitrary growing-block unitary must not be counted as a free matching gate.

For GHZ seeds a permutation of sector labels and matching amplitude magnitudes allow relative phases to be corrected on a single encoded register. This observation is not used as an unproved circuit assumption: the conversion theorem requires the actual circuit R and counts its physical-site depth. General canonical data do not imply that matching. Complex weights, multiplicities and length dependence must be retained; when using the corrected one-copy construction, its corrected input amplitudes must be used, not silently replaced by the printed coefficients.

References

- [MSWC24] Daniel Malz, Georgios Styliaris, Zhi-Yuan Wei, and J. Ignacio Cirac. Preparation of matrix product states with log-depth quantum circuits. *Phys. Rev. Lett.*, 132:040404, 2024.