

Circuits Between States in the Same Phase: the Normal Case

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Verdict: scope-restriction (open).

1 What the source states

The paragraph “Discussion and outlook” of [MSWC24] states: “Our results also imply that MPS in the same phase can be transformed into each other using a log-depth circuit”. No proof is given. The results it refers to are the preparation of normal states in depth $O(\log(N/\varepsilon))$, eq. (1), and the preparation of all translation-invariant states, “including long-range non-normal ones”, with measurements. The main text recalls that “normal MPS all lie in the topologically trivial phase”.

2 What is formalized

Let A and B be normal tensors with the same physical dimension, and write $\hat{\phi}_N(A) = |\phi_N(A)\rangle/\|\phi_N(A)\|$ for the normalized periodic states and $\varepsilon(\psi, \phi) = 1 - |\langle\psi|\phi\rangle|$ for the error of eq. (1). There is c , depending only on A and B , such that for $N \geq 2$, $0 < \varepsilon \leq 1$ and nonvanishing periodic states, a local circuit U of depth at most $c \log(N/\varepsilon)$ satisfies $\varepsilon(U\hat{\phi}_N(A), \hat{\phi}_N(B)) \leq \varepsilon$; if both bond dimensions are positive, the same holds for all $N \geq N_0$ without the nonvanishing condition.¹ The circuit is $U_B U_A^\dagger$, with U_A and U_B the preparations of eq. (1) from the all-zero product state, each with error $\varepsilon/4$. The error is not a metric; the proof uses $\varepsilon(x, z) \leq 2(\varepsilon(x, y) + \varepsilon(y, z))$ for unit vectors, since $2\varepsilon(x, y)$ is the squared distance from x to the nearest unit multiple of y .

3 The restriction

Only the trivial phase of the classification without symmetry is covered: both tensors are normal. The normal-state theorem does not treat non-normal states

¹Formal declarations: `MPSPreparation.exists_isLocalCircuitOfDepth_le_log_of_mpvS`
`tate_ne_zero` and `MPSPreparation.exists_isLocalCircuitOfDepth_le_log` in `TNLean/MPS`
`/Preparation/CircuitEquivalence`.

or phases defined with a symmetry. Compatible coherent seeds give a sufficient non-normal conversion statement, while unequal GHZ sector probabilities obstruct chosen-vector conversion even for identical parent Hamiltonians; see [con26]. A complete interpretation requires a non-normal phase predicate specifying how the selected vector and its sector probabilities are transported.

References

- [con26] TNLean contributors. Compatible coherent seeds and non-normal state conversion. Technical report, TNLean project, 2026. https://sirui-lu.com/TNLean/paper-gaps/mswc24_same_phase_compatible_seed.pdf.
- [MSWC24] Daniel Malz, Georgios Styliaris, Zhi-Yuan Wei, and J. Ignacio Cirac. Preparation of matrix product states with log-depth quantum circuits. *Phys. Rev. Lett.*, 132:040404, 2024.