

A Corrected Approximating State for a Tensor with Repeated Blocks

2026-10-02

Verdict: local-correction (resolved).

1 What the source states

Let $A^i = \bigoplus_{j=1}^b \text{diag}(\mu_{j,1}, \dots, \mu_{j,m_j}) \otimes A_j^i$ with normal A_j [MSWC24, eq. (S2)], so that $|\phi_N\rangle \propto \sum_j \beta_j |v_j\rangle$ with $\beta_j = \sum_{k=1}^{m_j} \mu_{j,k}^N$ and $|v_j\rangle$ the periodic state of A_j [MSWC24, eqs. (S3), (S4)]. Block q sites and write the polar decomposition of the blocked map as $B = VP$. The source asserts $P = \bigoplus_j \text{diag}(\mu_{j,1}^q, \dots, \mu_{j,m_j}^q) \otimes P_j$ [MSWC24, eq. (S5)], replaces each P_j by its fixed point, and approximates the state on $N = qM$ sites by

$$|\tilde{\phi}_N\rangle \propto V^{\otimes M} \sum_{j=1}^b \beta_j |\Omega_j\rangle, \quad |\Omega_j\rangle = \bigotimes_{k=1}^M |\omega_j\rangle_{R_k L_{k+1}} \quad (1)$$

[MSWC24, eq. (S7)]. When some $m_j \geq 2$ the block form fails and no product of nearest-neighbour pairs with the weights β_j gives an approximation [con26a]. This note gives the corrected block form and the corrected approximating state, and shows that part (ii) of the approximation-error lemma holds for it when the q -site states of distinct blocks are orthogonal. When they overlap, it proves the bound at the rate of [con26b], for all nonzero complex weights and with a constant independent of the weights [con26c].

2 The corrected block form

Let $E_{j,k}$ be the isometry placing the copy k of block j in the bond space, so that $A^i = \sum_{j,k} \mu_{j,k} E_{j,k} A_j^i E_{j,k}^\dagger$ with $E_{j,k}^\dagger E_{j',k'} = \delta_{jj'} \delta_{kk'} \mathbf{1}$, and put $K_{j,k} = E_{j,k} \otimes E_{j,k}$. For $q \geq 1$ every word of length q is a direct sum, so the blocked map of A is

$$B = \sum_{j,k} \mu_{j,k}^q B_j K_{j,k}^\dagger = \sum_j c_j B_j L_j^\dagger, \quad c_j = \left(\sum_k |\mu_{j,k}|^{2q} \right)^{1/2}, \quad L_j = \sum_k \frac{\overline{\mu_{j,k}^q}}{c_j} K_{j,k}, \quad (2)$$

with B_j the blocked map of A_j , whenever some $\mu_{j,k} \neq 0$. The maps L_j are isometries with orthogonal ranges, $L_j^\dagger L_{j'} = \delta_{jj'}$, because $\sum_k |\mu_{j,k}^q|^2 / c_j^2 = 1$. If the q -site states of distinct blocks are orthogonal, $B_j^\dagger B_{j'} = 0$ for $j \neq j'$, the polar decomposition of an orthogonal sum [con26b] gives

$$V = \sum_j V_j L_j^\dagger, \quad P = \sum_j c_j L_j P_j L_j^\dagger, \quad (3)$$

with $B_j = V_j P_j$. On the copies of block j the positive part is $c_j w_j w_j^\dagger \otimes P_j$ with the unit vector $w_{j,k} = \overline{\mu_{j,k}^q} / c_j$: it has rank one in the copy index, where the source prints $\text{diag}(\mu_{j,1}^q, \dots, \mu_{j,m_j}^q)$. For $A^0 = \text{diag}(1, 1, 0)$, $A^1 = \text{diag}(0, 0, 1)$ this is the matrix $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ found in [con26a].

3 The corrected approximating state

Read $X = c_j L_j Y L_j^\dagger$ as a tensor with physical leg the output pair and virtual legs the input pair. Its entries are $X_{(ka),(kb)}^{(k'a',k'b')} = c_j w_{j,k'} \overline{w_{j,k}} Y_{ab}^{a'b'}$ and vanish off the copy-diagonal virtual pairs, so along a ring of M sites the virtual copy index is constant and

$$|\phi_M(X)\rangle = c_j^M \left(\sum_k \overline{w_{j,k}}^M \right) L_j^{\otimes M} |\phi_M(Y)\rangle = \beta_j L_j^{\otimes M} |\phi_M(Y)\rangle,$$

since $c_j^M \overline{w_{j,k}}^M = \mu_{j,k}^{qM}$. With $Y = P_j$ this gives the exact identity $|\phi_N\rangle \propto V^{\otimes M} \sum_j \beta_j L_j^{\otimes M} |v_{\text{pos},j}\rangle$, the corrected form of the display after eq. (S5) of the source, and replacing P_j by its fixed point gives the corrected approximating state

$$|\tilde{\phi}_N\rangle \propto V^{\otimes M} \sum_j \beta_j L_j^{\otimes M} |\Omega_j\rangle, \quad (4)$$

where $|\Omega_j\rangle$ is the product of the pairs ω_j of the fixed point of A_j in the bond space of block j . The weights β_j are those of the source; what changes is the fixed-point state: the isometry L_j acts on the two legs $L_k R_k$ of each site, so for $m_j \geq 2$ the state $L_j^{\otimes M} |\Omega_j\rangle$ carries the copy index on each site and is not a product of nearest-neighbour pairs of the full bond space. It is still obtained from a product of pairs by one on-site isometry per site. The states $L_j^{\otimes M} |\Omega_j\rangle$ are orthonormal, so the fixed-point state has norm $(\sum_j |\beta_j|^2)^{1/2}$ as in the source. For $m_j = 1$ and $\mu_j > 0$, $L_j = K_j$ and (4) is the source's state (1).

Since $V L_j = V_j$ by (3),

$$V^{\otimes M} \sum_j \beta_j L_j^{\otimes M} |\Omega_j\rangle = \sum_j \beta_j V_j^{\otimes M} |\Omega_j\rangle, \quad (5)$$

the combination, with the source's weights, of the approximating states of the normal case for the blocks. Equivalently, the source's state (1) with each pair on one copy k_j becomes correct after replacing V by $V \sum_j L_j K_{j,k_j}^\dagger$.

4 The error bound

Assume $B_j^\dagger B_{j'} = 0$ for $j \neq j'$ and $\beta \neq 0$. This does not force $\phi_N \neq 0$, since a normal block can have $\phi_N(A_j) = 0$ at small N (for instance the Pauli matrices divided by $\sqrt{3}$ at $N = 1$). The target is $\sum_j \beta_j |v_j\rangle$ and, by (5), the corrected state is $\sum_j \alpha_j |s_j\rangle$ with $\alpha_j = \beta_j / (\sum_l |\beta_l|^2)^{1/2}$ and $|s_j\rangle = V_j^{\otimes M} |\Omega_j\rangle$. All cross overlaps vanish, as for multiplicity one [con26b], so with $p_j = |\beta_j|^2$, $z_j = \langle s_j | v_j \rangle$ and $c'_j = \langle v_j | v_j \rangle$,

$$|\langle \tilde{\phi}_N | \phi_N \rangle| = \frac{|\sum_j p_j z_j|}{(\sum_j p_j)^{1/2} (\sum_j p_j c'_j)^{1/2}},$$

and $\varepsilon \leq 2 \sum_j (|z_j - 1| + |c'_j - 1|)$. Part (i) of the lemma for each block bounds this by $Cy e^{Cy}$ with $y = Me^{-\gamma q / \xi_{\text{diag}}}$, which is $(N/q)e^{-\gamma q / \xi_{\text{diag}}}$ for $q \geq 1$, with no condition $q = o(N)$. The phases and the possible cancellations in β_j enter only through the weights p_j .

5 Numerical check

A script evaluates both states directly from the singular value decomposition of B , for $N = qM$.

- For $A^0 = \text{diag}(1, 1, 0)$, $A^1 = \text{diag}(0, 0, 1)$ the corrected state has overlap 1.000000 with ϕ_N for $q = 1, 2$ and $M = 1, 2, 3, 4, 6$, where the source's state with the pair on one copy gives 0.988496, 0.948683, 0.881546, 0.800000, 0.650791, and the pair spread over both copies gives 1.000000, 0.948683, 0.800000, 0.650791, 0.502136.
- For $d = 4$, a block of bond dimension two on the physical indices $\{0, 1\}$ with transfer eigenvalues of moduli 1, 0.2421, 0.2421, 0.1929 and copy weights $(1, 0.8e^{0.7i})$, and a block of bond dimension one on $\{2, 3\}$ with weight 0.9, the corrected state gives 0.999321, 0.999800, 0.999644 at $q = 3$ and $M = 1, 2, 3$, and 0.997257, 0.995544, 0.993003, 0.990941 at $q = 2$ and $M = 2, 3, 4, 5$, where the source's state with the pair on one copy gives 0.997866, 0.994090, 0.992242 and 0.983644, 0.966207, 0.954168, 0.935826. In every case the squared norm of the corrected fixed-point state equals $\sum_j |\beta_j|^2$.
- For the overlapping blocks $A_1 = (1, 0)$, $A_2 = (3/5, 4/5)$ of [con26b] the corrected state is the source's state, and both give 0.857493, 0.771528, 0.713544 at $q = 1$ and $M = 2, 4, 6$: the correction concerns multiplicities, not overlaps.

6 Blocks that occur

The decomposition (S2) lists the normal tensors that occur in A , each copy with its weight. A block with no copies, or a copy whose weight $\mu_{j,k}$ vanishes, is a summand with coefficient zero: it contributes nothing to A and only pads the bond dimension, and a block with no nonzero weight has $c_j = 0$ and L_j undefined. The formalization adopts the intended reading as a convention of the data: every block has at least one copy, and every copy has a nonzero weight. This is a local fix of a degenerate reading, not a hypothesis added to the source.

7 What is formalized

The corrected block form (3), the identity (5) and the overlap formula are formalized for arbitrary multiplicities and nonzero complex weights, for blocks whose q -site states are orthogonal, at every $q \geq 1$; the overlap formula in addition needs every blocked tensor B_j to be injective, so that the vectors $|s_j\rangle$ are unit vectors, and the error bound covers the shorter block lengths by $\varepsilon \leq 1$ after enlarging the constants. The bound $\varepsilon \leq Cy e^{Cy}$, together with its O -form $\varepsilon \leq Cy$, is formalized under the same assumptions, with $y = Me^{-\gamma q/\xi}$, at every $q \geq 0$ and $M \geq 1$ with $\beta \neq 0$.¹ The orthogonality restriction is the one recorded in [con26b], where for two overlapping blocks of multiplicity one the error lies between $\tau^{2q}/16$ (for $N/q \geq 3$) and $(N/q)\tau^{2q}$, with τ the eigenvalue of their mixed transfer matrix, which the source's rate does not control; the construction here does not change that example. For blocks of multiplicity one and weight one whose states overlap, that note proves the bound with the correlation lengths of the mixed transfer matrices included in the rate. The norm of the fixed-point state is formalized only as the bound one after $V^{\otimes M}$; the overlap normalizes the state after $V^{\otimes M}$.

8 Repeated blocks whose states overlap

Drop the orthogonality of the q -site states of distinct blocks, let the blocks be normal in the gauge $\sum_i (A_j^i)^\dagger A_j^i = \mathbf{1}$ with positive definite fixed points σ_j of trace one, let λ_2 bound the moduli of the eigenvalues other than one of every E_{jj} and of all eigenvalues of the mixed transfer matrices $E_{jj'}$, $j \neq j'$, let $\xi = -1/\ln|\lambda_2|$ and $0 < \gamma < 1$. The corrected state (4) still converges, at the rate of [con26b]: there are C, C' , independent of the weights, with

$$\varepsilon \leq Cy e^{Cy}, \quad \varepsilon \leq C'y, \quad y = Me^{-\gamma q/\xi}, \quad (6)$$

¹Formal declarations: `MPSTensor.polarIso_blockTensor_repeatedBlockSum`, `MPSTensor.polarPos_blockTensor_repeatedBlockSum` and `MPSTensor.copyApproxVector_repeatedBlockSum` in `TNLean/MPS/Preparation/RepeatedBlockSum`; `MPSTensor.exists_approximationError_le_repeatedBlockSum` and `MPSTensor.exists_approximationError_le_mul_repeatedBlockSum` in `TNLean/MPS/Preparation/BlockSumError`.

for all nonzero complex weights, every $q \geq 0$ and $M \geq 1$ with $\beta \neq 0$. The proof below first gives the weaker bound $\varepsilon \leq Cy e^{Cy} / \min(1, b)^{1/2}$, $b = \sum_j |\beta_j|^2$, for $|\mu_{j,k}| \leq 1$; the relative estimate of [con26c] removes the factor and the condition on the weights.

The proof combines the two arguments. By (2) the blocked map is $B = \sum_j c_j B_j L_j^\dagger$ with $c_j \leq m_j^{1/2}$, so the blocks of $B^\dagger B$ are $c_j c_{j'} L_j B_j^\dagger B_{j'} L_{j'}^\dagger$, and the Gram estimate of [con26b], which uses only that the embeddings have norm at most one, gives $\|B^\dagger B - \sum_j c_j^2 L_j (\sigma_j^T \otimes \mathbf{1}) L_j^\dagger\| \leq K e^{-2\gamma q/\xi}$. The $\frac{1}{2}$ -Hölder bound of the square root gives $\|P - P_\infty\| \leq K' e^{-\gamma q/\xi}$ with $P_\infty = \sum_j c_j L_j (\sqrt{\sigma_j}^T \otimes \mathbf{1}) L_j^\dagger$, whose square is the limit of the Gram matrices since the L_j are isometries with orthogonal ranges. Since $V^\dagger B = P$, the overlap of the unnormalized state $V^{\otimes M} \sum_j \alpha_j L_j^{\otimes M} |\Omega_j\rangle$ with $|\phi_N(A)\rangle$ is $b^{-1/2} \sum_j \bar{\beta}_j z_j$, with $z_j = \langle \Omega_j | \phi_M(L_j^\dagger P) \rangle$, where $L_j^\dagger P$ is read as a tensor whose physical leg is a bond pair of block j . The row $L_j^\dagger P_\infty$ is $\sum_k \mu_{j,k}^q (\sqrt{\sigma_j}^T \otimes \mathbf{1}) K_{j,k}^\dagger$, and its mixed transfer matrix against the fixed-point tensor of σ_j is $\sum_k \mu_{j,k}^q R_k$ with $R_k(\rho) = \text{Tr}(E_{j,k}^\dagger \rho) E_{j,k} \sigma_j$. The R_k are orthogonal idempotents of trace one, so the powers $\sum_k \mu_{j,k}^{qn} R_k$ are bounded uniformly in the weights and the M -th power has trace β_j ; the telescoping estimate gives $|z_j - \beta_j| \leq C_j y e^{C_j y}$. The states $L_j^{\otimes M} |\Omega_j\rangle$ are orthonormal and $V^{\otimes M}$ does not increase norms, so normalizing the approximating state only increases the overlap. Finally $\| |\phi_N(A)\rangle \|^2 - b \leq b K'' e^{-\gamma N/\xi}$, since every coefficient $\beta_j \bar{\beta}_{j'}$ has modulus at most b . With $S = \sum_j \bar{\beta}_j z_j$ one has $|S/b - 1| \leq b^{-1/2} \sum_j |z_j - \beta_j|$, and the inequality of [con26b] applied to S/b and $\|\phi_N(A)\|/b^{1/2}$ gives the weaker bound.

That bound exceeds (6) by the factor $\min(1, b)^{-1/2}$ when $b < 1$, for instance when the sums $\beta_j = \sum_k \mu_{j,k}^N$ cancel or every weight has modulus below one, because the estimates of the z_j are absolute. The relative estimate of [con26c] writes $z_j = \sum_{j'} \beta_{j'} w_{jj'}$ with $w_{jj'} = \delta_{jj'}$ up to $C_{jj'} y e^{C_{jj'} y}$, uniformly in the weights, from the bound $\|(P - P_\infty)C^{-1}\| \leq K e^{-\gamma q/\xi}$ with $C = \sum_j c_j L_j L_j^\dagger$; then $|S/b - 1| \leq \sum_{j,j'} C_{jj'} y e^{C_{jj'} y}$ with no factor, and (6) follows. The numerical check below shows the error converging at the same rate when b is exponentially small.

9 Numerical check with overlaps

A script evaluates the corrected state for two random normal blocks of bond dimension two on $d = 2$, with subleading eigenvalues 0.515 and 0.850 and mixed spectral radius 0.757, so that $|\lambda_2| = 0.850$, from the eigendecomposition of $B^\dagger B$, using $V^\dagger B = P$.

- For copy weights $(1, 0.8e^{0.7i})$ on the first block and 0.9 on the second, and $M = 8$, the error is 0.143, $7.56 \cdot 10^{-3}$, $3.95 \cdot 10^{-4}$ at $q = 4, 8, 12$, and $\varepsilon/(M|\lambda_2|^q)$ is $3.44 \cdot 10^{-2}$, $3.48 \cdot 10^{-3}$, $3.49 \cdot 10^{-4}$.
- For copy weights $(1, 1)$ and 1, and $M = 8$, the corrected state gives 0.241,

$3.49 \cdot 10^{-2}$, $6.75 \cdot 10^{-3}$ at $q = 4, 8, 12$, while the source's state with the pair on one copy gives 0.701, 0.499, 0.458.

- For copy weights $(1, -1)$ and 0.5 at odd N , where $\beta_1 = 0$, and $M = 5$, the error is 0.424, $7.54 \cdot 10^{-2}$, $1.51 \cdot 10^{-2}$ at $q = 5, 9, 13$, while $b = 0.5^{2N}$ is $8.9 \cdot 10^{-16}$, $8.1 \cdot 10^{-28}$, $7.3 \cdot 10^{-40}$.

The squared norm of the target is computed from $\sum_{j,j'} \overline{\beta_j} \beta_{j'} \langle \phi_N(A_j) | \phi_N(A_{j'}) \rangle$; evaluating it as the trace of the N -th power of the transfer matrix of A loses all digits when b is small.

10 What is formalized with overlaps

The bound (6), for all nonzero complex weights with constants independent of the weights, is formalized, with the ingredients above and the relative estimate of [con26c].² The weaker bound with the factor $\min(1, b)^{-1/2}$ is not formalized separately.

11 Copies with weights of different phases

When a block has several copies whose weights have different phases, the phases cannot be moved into the block as for multiplicity one [con26b], and the question is which coefficients the approximating state needs. For the corrected state (4) they are the coefficients $\beta_j = \sum_k \mu_{j,k}^N$ of the source: the isometry L_j carries the phases $\overline{\mu_{j,k}^q}/c_j$ of the copies, and the bounds of the previous sections hold for complex weights. The sums $\sum_k |\mu_{j,k}|^N$, which give the coefficient $|\mu_j|^N$ of multiplicity one, and the copy norms c_j^M do not work in their place. Take $d = 2$, the block $A_1 = (1, 0)$ with copy weights $(1, -1)$ and the block $A_2 = (0, 1)$ with weight one. For odd N , $\beta = (0, 1)$ and $|\phi_N\rangle \propto |1 \cdots 1\rangle$, and the corrected state with the coefficients $(2, 1)$ is proportional to $2|0 \cdots 0\rangle + |1 \cdots 1\rangle$, so $\varepsilon = 1 - 1/\sqrt{5}$ for every odd q and M .

The source's construction, with the pairs of block j on one copy k_j , also converges once its coefficients are changed. For $q \geq 1$ the blocked map is $B = \sum_j c_j B_j L_j^\dagger$ by (2), and $L_{j'}^\dagger K_{j,k} = \delta_{jj'} (\mu_{j,k}^q / c_j)$, so $B K_{j,k} = \mu_{j,k}^q B_j = (\mu_{j,k}^q / c_j) B L_j$. The partial isometry V vanishes on the kernel of B : if $BY = 0$

²Formal declarations: `MPSTensor.exists_approximationError_le_repeatedOverlappingBlockSum` and `MPSTensor.exists_approximationError_le_mul_repeatedOverlappingBlockSum` in `TNLean/MPS/Preparation/RepeatedOverlappingBlockWeights`; `MPSTensor.sum_star_copyApproxVector_mul_mpv` and `MPSTensor.sum_norm_sq_copyApproxVector_le` in `TNLean/MPS/Preparation/RepeatedOverlappingBlockError`; `MPSTensor.exists_norm_m_polarPos_blockTensor_repeatedBlockSum_sub_le`, `MPSTensor.mixedMapLM_copyPosLimitRow_apply` and `MPSTensor.exists_norm_mpvOverlap_sub_bntWeight_le_of_norm_sub_copyPosLimitRow_le` in `TNLean/MPS/Preparation/RepeatedOverlappingBlockOverlap`; `MPSTensor.exists_norm_gram_sum_sub_le` and `Matrix.norm_polarPos_sub_le_sqrt` in `TNLean/MPS/Preparation/OverlappingBlockGram`.

then $PY = 0$, hence $\Pi Y = 0$ for the support projector Π of P , and $VY = V\Pi Y = 0$. Therefore

$$VK_{j,k} = \frac{\mu_{j,k}^q}{c_j} VL_j, \quad (7)$$

for blocks whose q -site states overlap as well, and $V^{\otimes M} K_{j,k_j}^{\otimes M} |\Omega_j\rangle = (\mu_{j,k_j}^q / c_j)^M V^{\otimes M} L_j^{\otimes M} |\Omega_j\rangle$. The source's state (1) with the pairs on the copies k_j and the coefficients

$$\beta'_j = \beta_j \left(\frac{c_j}{\mu_{j,k_j}^q} \right)^M \quad (8)$$

is the corrected state (4). For $m_j = 1$, $c_j = |\mu_j|^q$ and $\beta'_j = |\mu_j|^N$, the coefficient that repairs (S7) for multiplicity one [con26b]; for the tensor $A^0 = \text{diag}(1, 1, 0)$, $A^1 = \text{diag}(0, 0, 1)$ of [con26a], $\beta' = (2 \cdot 2^{M/2}, 1)$, and the state is exact. The coefficients β'_j depend on q and M separately, not only on N . With the printed coefficients β_j the state (1) fails, for $m_j \geq 2$ already with positive weights [con26a], and for $m_j = 1$ already for two blocks with the weights 1 and $\mu \neq 1$, $|\mu| = 1$ [con26b].

12 Numerical check with phases

The script of the previous numerical checks evaluates both constructions for the two random blocks used there, with copy weights $(1, e^i)$ on the first block and 1 on the second, at $M = 5$ and $q = 3, 5, 7, 9, 11, 13$.

- The corrected state with $\beta_j = \sum_k \mu_{j,k}^N$ gives $0.477, 9.75 \cdot 10^{-2}, 9.74 \cdot 10^{-2}, 1.67 \cdot 10^{-2}, 9.35 \cdot 10^{-3}, 6.94 \cdot 10^{-3}$.
- The corrected state with $\sum_k |\mu_{j,k}|^N$ gives $0.538, 9.77 \cdot 10^{-2}, 0.440, 3.83 \cdot 10^{-2}, 7.30 \cdot 10^{-2}, 0.187$, and with c_j^M it gives $0.549, 0.114, 0.558, 7.73 \cdot 10^{-2}, 0.122, 0.261$.
- The source's state with the pairs on the first copy gives, with $\beta_j = \sum_k \mu_{j,k}^N$, $0.614, 0.403, 0.159, 0.295, 0.258, 0.177$, and with $\sum_k |\mu_{j,k}|^N$, $0.609, 0.403, 0.195, 0.291, 0.254, 0.188$.
- With the coefficients (8) the source's state gives the errors of the corrected state. Over three sets of weights, $q \in \{1, 2, 3, 5\}$, $M \in \{1, 3, 4\}$ and every choice of the copies k_j , the two errors agree to $6 \cdot 10^{-8}$, except for the copy of weight $i/2$ in the set $((1, -1, i/2), (0.6))$, where the factor $(c_j / \mu_{j,k_j}^q)^M$ reaches 10^5 and they agree to $6 \cdot 10^{-6}$; (7) holds to 10^{-14} .

13 What is formalized with phases

The identity (7), the equality of the overlaps of the source's state with the coefficients (8) and of the corrected state, the value $\beta'_j = |\mu_j|^N$ for a block with

one copy, and the error bounds for the source's state with these coefficients are formalized, for complex weights of any phase: for blocks whose q -site states are orthogonal, $\varepsilon \leq Cye^{C'y}$ and $\varepsilon \leq C'y$ at every $q \geq 1$ and $M \geq 1$ with $\beta \neq 0$, and for overlapping blocks the bound (6) at every $q \geq 1$, for all nonzero weights with constants independent of the weights.³ The failure of the coefficients $\sum_k |\mu_{j,k}|^N$ and c_j^M is checked numerically and not formalized.

References

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³Formal declarations: `Matrix.polarIso_mul_eq_zero` in `TNLean/MPS/Preparation/PolarUniqueness`; `MPSTensor.polarIso_blockTensor_repeatedBlockSum_mul_pairEmbedding`, `MPSTensor.oneCopyWeight`, `MPSTensor.oneCopyWeight_of_subsingleton`, `MPSTensor.noNormalApproxOverlap_repeatedBlockSum_oneCopy`, `MPSTensor.exists_approximationError_le_repeatedOverlappingBlockSum_oneCopy` and `MPSTensor.exists_approximationError_le_mul_repeatedOverlappingBlockSum_oneCopy` in `TNLean/MPS/Preparation/OneCopyPairState`; `MPSTensor.exists_approximationError_le_repeatedBlockSum_oneCopy` and `MPSTensor.exists_approximationError_le_mul_repeatedBlockSum_oneCopy` in `TNLean/MPS/Preparation/BlockSumError`.