

Long-Range Matrix Product States with Measurements: Scope of the Formal Statement

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Verdict: scope-restriction (resolved).

1 What the source states

The paragraph “Long-range MPS using measurements” of [MSWC24] writes the fixed point of a translation-invariant MPS that is not normal, up to an isometry, as

$$|\Omega'\rangle = \sum_{j=1}^b \alpha_j^{(N)} \bigotimes_{i=1}^{N/q} |\omega_j\rangle_{R_i L_{i+1}} = W^{\otimes N/q} |\chi_{N/q}\rangle, \quad |\chi_M\rangle = \sum_{j=1}^b \alpha_j |j\rangle^{\otimes M}, \quad (1)$$

with $W: |j\rangle \mapsto |\omega_j\rangle$, and prepares it as follows: “First create $|\chi_{N/q}\rangle$, which can be done in constant depth with measurements ... Subsequently, apply in parallel the isometries W ..., which also takes constant depth.” With the approximation of the target by a state of the form (1) up to local isometries, obtained by blocking $q \propto \log(N/\varepsilon)$ sites, it concludes that if “measurements are only used for the preparation of $|\chi_{N/q}\rangle$, the depth is $O(\log(N/\varepsilon))$ ”, for all translation-invariant MPS, short- or long-range correlated.

2 The statements formalized

Orthogonal blocks and block lengths dividing N . Let $A^i = \bigoplus_j \text{diag}(\mu_{j,1}, \dots, \mu_{j,m_j}) \otimes A_j^i$ with normal blocks A_j in the gauge of eq. (5) of the source and nonzero weights. The first formal statement prepares, with measurements, a unit vector of error at most ε in depth at most $c \log(N/\varepsilon)$, for $N \geq 2$, $0 < \varepsilon \leq 1$, and block lengths q with $a \log(N/\varepsilon) + b' \leq q \leq 2(a \log(N/\varepsilon) + b')$ under three restrictions.¹

¹Formal declarations: `MPSPreparation.exists_isPreparedWithMeasurementsAndCircuitInDepth_le_log_repeatedBlockSum` and `MPSPreparation.exists_isPreparedWithMeasurementsAndCircuitInDepth_copyApproxVector` in `TNLean/MPS/Preparation/NonNormalMeasurementPreparation`.

1. The q -site states of distinct blocks are orthogonal. Without this the block form of the positive part fails [con26b], and the isometry V of the blocked direct sum is no longer the sum of the isometries V_j of the blocks, which is what the block unitary of this statement implements.
2. The block length q divides N , so that the chain consists of N/q blocks of q sites, as in eq. (10) of the source. Together with the window above, this statement applies only to chain lengths N with a divisor in this window, and it is vacuous for the others, for instance for every large prime N .
3. The state prepared is the corrected approximating state of [con26a], which agrees with the state of eq. (S7) when every multiplicity is one and every weight is positive.

Overlapping blocks and every chain length. The second formal statement removes the three restrictions above.² For A as above, with every fixed point positive definite and the mixed transfer maps of distinct blocks of spectral radius below one, there is c such that for every $N \geq 2$ and $0 < \varepsilon \leq 1$ at which $|\phi_N(A)\rangle \neq 0$, a unit vector of error at most ε is prepared with measurements and a circuit in depth at most $c \log(N/\varepsilon)$. There is N_0 such that $|\phi_N(A)\rangle \neq 0$ for every $N \geq N_0$ at which the weights $\beta_j = \sum_k \mu_{j,k}^N$ are not all zero. The q -site states of distinct blocks may overlap, the multiplicities and the complex weights are arbitrary, and the chain is cut into blocks “all of the same size, q_N , except for the last one, which may be larger”, as in the proof of Theorem 1 of the source. The statement reads the construction as follows.

1. The isometries applied on the blocks are the isometric factors $V^{(\ell)}$ of the polar decompositions of the blocked maps of the direct sum $\bigoplus_j A_j$ with weights one and one copy of each block, not that of the blocked map of A , whose positive part does not have the block form (S5) when the blocks overlap or repeat. For large ℓ this blocked map is injective on the bond pairs of the blocks, by the positive definiteness of the limit of its Gram matrix, so $V^{(\ell)}$ is an isometry on these pairs. The state prepared is

$$|\psi\rangle = \sum_j \alpha_j \left(\bigotimes_k V^{(\ell_k)} \right) \bigotimes_k |\omega_j\rangle, \quad \alpha_j = \frac{\beta_j}{(\sum_l |\beta_l|^2)^{1/2}},$$

with ω_j the pair of the fixed point of block j . The weights enter only through the amplitudes, so the overlap $\langle \psi | \phi_N(A) \rangle$ is a quadratic form $\sum_{j,j'} \beta_j \beta_{j'} z_{jj'}$ divided by $(\sum_l |\beta_l|^2)^{1/2}$, with $|z_{jj'} - \delta_{jj'}| \leq C y e^{C y}$ and $y = M e^{-\gamma q/\xi}$. The error is therefore at most $C M e^{-\gamma q/\xi}$ with C independent

²Formal declarations: `MPSPreparation.exists_isPreparedWithMeasurementsAndCircuitInDepth_le_log_of_mpvState_ne_zero` and `MPSPreparation.exists_isPreparedWithMeasurementsAndCircuitInDepth_le_log` in `TNLean/MPS/Preparation/OverlappingMeasurementPreparation`.

of the weights, and the factor $\min(1, \sum_j |\beta_j|^2)^{-1/2}$ of the error bound of the corrected state [con26d] does not enter.³

2. The rate is $e^{-\gamma q/\xi}$ with ξ bounding the correlation lengths of the blocks and of the mixed transfer maps of distinct blocks [con26b].
3. The periodic state is assumed nonzero, as the normalization of the source presupposes [con26c]; this holds for $N \geq N_0$ when the weights β_j are not all zero.

For N below the block length the normalized target itself is prepared, as the state of one block or, below a fixed length, by a circuit of bounded depth.

3 Two readings of the construction

The circuit after the measurement. The model of preparation with measurements of [PSC21] applies a circuit, then measurements and local corrections. The construction of the source applies circuits after the preparation of $|\chi_{N/q}\rangle$. The formal statement counts such a preparation as a vector prepared with measurements in depth T_1 , followed by a local circuit of depth T_2 that does not depend on the outcomes, of total depth $T_1 + T_2$.

Constant depth on the physical chain. The one-round construction uses SWAP routing and has depth $O(q)$. With additional measurement rounds the GHZ-type state is prepared in constant depth on the original chain itself, for arbitrary cyclic block lengths $\ell_k \geq 3r_1$ and $r_1 \geq 2$.⁴ For an odd block, a unitary on its first $r_1 + 1$ sites moves the first register one site to the right, leaving an even scratch gap. The two registers are then joined by a teleportation-assisted controlled subtraction of depth $4r_1 + K + 2$, independent of their separation, and the padding rotation is undone. All stretches lie inside disjoint blocks, so they run in parallel. The same construction covers the singleton cyclic ring and arbitrary parity patterns. The original cyclic measurement and corrections finish the GHZ state, with an outcome scalar independent of the coherent label amplitudes.

4 Resolution and resource convention

The overlapping-block result removes the earlier orthogonality and chain-length restrictions. The sparse-register construction now removes the remaining GHZ depth restriction. It restores all scratch sites and ancillas exactly to $|0\rangle$, uses no postselection or additional physical sites, and counts all quantum layers

³Formal declaration: `MPSTensor.exists_one_sub_norm_inner_sum_blockIsometryState_le` in `TNLean/MPS/Preparation/OverlappingBlockStates`.

⁴Formal declarations: `MPSPreparation.exists_isPreparedWithMeasurementRoundsInDepth_windowGHZState` and `MPSPreparation.exists_rounds_sparseBlockShift` in `TNLean/MPS/Preparation/SparseWindowGHZ`.

on neighbouring sites. On-site measurements, on-site corrections and global classical feedforward are free in the measurement-round model. Neither an arbitrary unitary on q sites nor a hidden environment is treated as free.

References

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