Two-Projection Decomposition: Repeated-Projector Typo

2026-08-12

Verdict: local-correction (resolved).

1 Source passage

In the proof of Proposition IV.5 of [CPGSV17], the two-projection lemma is used to decompose two projectors on $\mathbb{C}^D$. The source writes

$$P = \bigoplus_i |0\rangle\langle 0|_i \oplus R, \qquad P = \bigoplus_i |v_i\rangle\langle v_i|_i \oplus S \tag{1}$$

at lines 760–763 of the local source file. The second projector must be Q, not P. This follows from the preceding sentence, which introduces projectors P and Q, and from the subsequent identities involving PQ and QP.

2 Formalized interpretation

The generic angle-block declarations `LinearMap.IsSymmetricProjection.angleDefect_block` and `LinearMap.IsSymmetricProjection.exists_orthonormal_angle_block` therefore use symmetric projections P and Q. They prove the exact local action of both operators on a generic two-dimensional block. The declarations `LinearMap.IsSymmetricProjection.angleDefectSum` and `LinearMap.IsSymmetricProjection.angleDefectComplement` assemble the compression-indexed blocks into an orthogonal internal sum and its invariant orthogonal complement. On that complement, the restricted projections commute. This is a local correction of a typographical error and does not add a hypothesis or alter the mathematical claim.

The declaration `LinearMap.IsSymmetricProjection.reducedProjection` defines the reduced projector as the orthogonal projector onto $\mathrm{ran}(PQ)$. The source defines its projector blockwise; identifying that projector with the canonical construction remains part of issue #6294. The associated theorems prove the three absorption identities in lines 767–769, the range identities

$$\operatorname{ran}(\widetilde{P}) = \operatorname{ran}(PQ), \tag{2}$$

$$\operatorname{ran}(PQP) = \operatorname{ran}(PQ), \tag{3}$$

$$\operatorname{ran}(\widetilde{P}Q) = \operatorname{ran}(\widetilde{P}). \tag{4}$$

The final range identity is the derived input to the equal-range right-factor theorem, which gives the invertible map from line 770.

3 Verdict

Verdict: resolved local fix. The formalized angle block uses Q as the second projector, which resolves the source's repeated P label without changing the mathematical statement. The compression-indexed blocks, including endpoint coordinates, are assembled into an orthogonal internal sum with a commuting complementary remainder. The reduced projector and its identities from lines 764–770 are also formalized. The coordinate-free reduced projector has not been identified with the source's particular blockwise projector; no such identification is needed for its proved absorption and range identities.

References

[CPGSV17] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product unitaries: Structure, symmetries, and topological invariants. *Journal of Statistical Mechanics: Theory and Experiment*, 2017(8):083105, 2017.