

Identity-Ancilla Transport of MPU Symmetries

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Verdict: scope-restriction (open).

1 Source definition

Cirac–Pérez–García–Schuch–Verstraete define two MPU tensors to be equivalent under a symmetry when identity ancillas and a common blocking make them strictly equivalent under that symmetry [CPGSV17, lines 1356–1366]. The definition does not state how an action on the original physical dimension determines an action after adjoining an identity ancilla.

2 Missing transport

Let $d > 0$ be the original physical dimension, let $N > 0$ be the chain length, and let $x > 1$ be an ancilla dimension. The canonical sitewise rearrangement is

$$R_{d,x,N}: (\mathbb{C}^d)^{\otimes N} \otimes (\mathbb{C}^x)^{\otimes N} \longrightarrow (\mathbb{C}^{dx})^{\otimes N}. \quad (1)$$

It induces the operator embedding

$$\iota_x(X) = R_{d,x,N}(X \otimes I_{x^N})R_{d,x,N}^{-1}. \quad (2)$$

Under these nondegeneracy assumptions, the map in (2) is not onto the enlarged operator algebra. Consequently, an action at dimension d can constrain the enlarged action on $\iota_x(X)$ but cannot determine its values on arbitrary operators at dimension dx . A strict-equivalence path can leave the image of ι_x , so an action defined only on that image is insufficient.

The formalized restricted construction therefore takes actions at every physical dimension as supplied data and assumes

$$\iota_x(\mathcal{S}_{d,N}(X)) = \mathcal{S}_{dx,N}(\iota_x(X)). \quad (3)$$

The symmetry family supplies the condition in (3).¹

¹Formal declarations: `MPOTensor.FiniteChainOperatorSymmetryFamily` supplies the dimension-indexed actions and semiconjugacy law; `MPOTensor.EquivalentUnderSymmetry` defines the coherent-family equivalence.

3 Additional common ancillas

The preliminary equivalence convention chooses positive coprime ancilla sizes p_a, p_b with $p_a d_a = p_b d_b$ [CPGSV17, lines 706–724]. Later, the source compares two tensors of the same physical dimension by adjoining an identity ancilla of the same dimension d to each tensor [CPGSV17, lines 2210–2215]. For $d > 1$, this common pair is not coprime. In fact, when $d_a = d_b > 0$, dimension balance forces $p_a = p_b$, and coprimality then forces $p_a = p_b = 1$.

Thus the current coprime-ancilla equivalence predicates cannot express this later common-ancilla construction directly. This is a further restriction, independent of the missing symmetry extension in (3). A source-level stabilized equivalence must allow a common additional ancilla after the minimal coprime enlargement, or equivalently permit arbitrary positive balanced ancilla sizes. Its symmetry actions must be defined on the full enlarged operator algebra. The one-sided swap comparison of strict equivalence does not discharge any of these stabilization requirements.

4 Verdict

Verdict: scope restriction. The coherent-family definition is a well-defined strengthened variant, not the unrestricted source definition. The source-level definition remains unformalized until the intended actions on the full enlarged operator algebras are specified without adding the semiconjugacy hypothesis to the cited statement, and the common-ancilla construction is admitted without the minimal coprime-pair restriction.

References

- [CPGSV17] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product unitaries: Structure, symmetries, and topological invariants. *Journal of Statistical Mechanics: Theory and Experiment*, 2017(8):083105, 2017.