

Specified-Tensor Shift Values and the Public MPU Index

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Verdict: scope-restriction (resolved).

1 Source statement

For a simple matrix product unitary tensor with positive right and left source-cut ranks r and ℓ , Definition IV.1 of Ref. [CPGSV17] uses

$$\frac{1}{2}(\log_2 r - \log_2 \ell). \quad (1)$$

Proposition IV.2 proves that this value is independent of the chosen simple blocking. The resulting blocking-independent quantity is the public MPU index.

For the topological-insulator examples, the paper states that the right shift $T^{(N)}$ has index $\log_2 d$, its adjoint left shift $T^{(N)\dagger}$ has index $-\log_2 d$, and the three displayed families

$$U_1^{(N)} = (\mathbf{1} \otimes \mathbf{1})^{\otimes N}, \quad (2)$$

$$U_2^{(N)} = T^{(N)\dagger} \otimes T^{(N)}, \quad (3)$$

$$U_3^{(N)} = T^{(N)} \otimes T^{(N)\dagger} \quad (4)$$

have index zero.

2 Resolution

Let E be the transfer matrix of a normal tensor with bond dimension D , and let $(\Phi|$ and $|\rho)$ be its left and right eigenvectors for the eigenvalue one, regarded as $D \times D$ matrices. Following Ref. [CPGSV17], the tensor is in canonical form II when $(\Phi| = \sum_{n=1}^D (n, n|$ and $|\rho) = \sum_{n=1}^D \rho_n |n, n)$ with every $\rho_n > 0$ and $(\Phi|\rho) = 1$; that is, Φ is the identity matrix and ρ is a positive diagonal matrix of unit trace. A canonical-form-II representative of an MPU tensor is a tensor in this form generating the same periodic operators.

The source-rank calculations are now identified with the public, blocking-independent index. The normalized right-shift transfer map is $X \mapsto \text{tr}(X)I/d$, so the positive diagonal matrix I/d supplies a canonical-form-II representative. The existing source-index calculation then gives $\text{ind}(T) = \log_2 d$. Physical adjunction negates the index and tensor products add indices. Thus $\text{ind}(T^\dagger) = -\log_2 d$ and all three displayed families have index zero.

These conclusions are proved by `MPOTensor.rightShiftTensor_index`, `MPOTensor.leftShiftTensor_index`, `MPOTensor.shiftExampleU_index`, `MPOTensor.shiftExampleU_index`, and `MPOTensor.shiftExampleU_index` in `TNLean/MPS/MPU/Examples/ShiftPublicIndex.lean`. The canonical-form witness is `MPOTensor.rightShiftTensor_exists_canonicalFormII`.

The general theorem `MPOTensor.IsMPU.index_eq_of_mpo_eq` transports these values to any positive-dimensional MPU tensor generating the same periodic operators. No equality of the raw source ranks of arbitrary unreduced presentations is required or asserted.

3 Scope

The original numerical gap is resolved. The older specified-tensor calculations remain useful intermediate results, but the source’s public index claim no longer depends on treating them as the public index itself. The separate assertion that shifts cannot be approximated by short-range time evolution [CPGSV21] is not a consequence established by these numerical theorems.

References

- [CPGSV17] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product unitaries: Structure, symmetries, and topological invariants. *Journal of Statistical Mechanics: Theory and Experiment*, 2017(8):083105, 2017.
- [CPGSV21] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product states and projected entangled pair states: Concepts, symmetries, theorems. *Reviews of Modern Physics*, 93(4):045003, 2021.