Fixed Virtual Bond Dimension in MPU Tensor Equivalence

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Verdict: scope-restriction (open).

1 Source definition

Let $\mathcal{U}$ and $\mathcal{V}$ be matrix product unitary tensors with physical dimensions d_a and d_b, respectively, and let p_a and p_b be positive coprime ancilla dimensions. Cirac–Pérez-García–Schuch–Verstraete define strict equivalence by a continuous path of matrix product unitary tensors and define equivalence by first attaching the identity ancillas and then applying a common blocking length [CPGSV17, lines 706–724]. The source balances the enlarged physical dimensions through

$$p_a d_a = p_b d_b. \tag{1}$$

It does not give a corresponding stabilization or identification for unequal raw virtual bond dimensions.

2 Formal scope

The declarations `MPOTensor.StrictlyEquivalent` and `MPOTensor.Equivalent` formalize these two relations. A continuous path must lie in one ambient topological space, so both definitions fix a virtual bond dimension D along the path and at both endpoints. Different physical dimensions are compared only through an explicit equality and the induced reindexing of their physical coordinates. For the weaker equivalence relation, identity ancillas are attached before blocking, exactly as in the source.

Extending the definition to unequal raw virtual bond dimensions would require an additional mathematical stabilization convention not supplied in the cited passage.

3 Verdict

Verdict: scope restriction. The formal definitions are source-faithful in one fixed ambient virtual bond dimension. They neither assert nor require a relation

between tensors of unequal raw virtual bond dimensions. Any such relation must be introduced as a separately named stabilization convention rather than folded into the source definitions.

References

[CPGSV17] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product unitaries: Structure, symmetries, and topological invariants. *Journal of Statistical Mechanics: Theory and Experiment*, 2017(8):083105, 2017.