

Nonzero Weights in the MPU Canonical Form

2026-09-26

Verdict: clarification (resolved).

1 Canonical blocks and zero weights

The canonical form of Cirac–Pérez-García–Schuch–Verstraete writes each local matrix as

$$A^i = \bigoplus_{k=1}^r \mu_k A_k^i, \quad (1)$$

where each A_k is irreducible and its transfer map has spectral radius one [1, canonical-form definition]. The source does not explicitly say $\mu_k \neq 0$. A zero weight, however, contributes no matrix at any site and does not describe a canonical block of A . The formal canonical-form predicate therefore lists only weights satisfying $\mu_k \neq 0$.

This convention is needed in the proof of Proposition III.1. A retained irreducible block has a nonzero transfer eigenvalue after weighting only when $\mu_k \neq 0$. The shifted trace identities of an MPU then force that eigenvalue to equal one. With zero weights, arbitrary invisible blocks could be adjoined and the number of listed blocks would no longer be determined by the transfer spectrum.

Nonzero weights do not resolve unused ambient bond coordinates. The separate full-support convention requires $\sum_k D_k = D$ for the MPU canonical-form representative; its mathematical scope and the distinction from literal CPSV canonical-form data are recorded in https://sirui-lu.com/TNLean/paper-gaps/mpu_canonical_form_full_support.pdf.

References

- [1] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product unitaries: Structure, symmetries, and topological invariants. *Journal of Statistical Mechanics: Theory and Experiment*, 2017(8):083105, 2017.