The Endpoint Rank-Product Exponent in MPU Index Well-Definedness

2026-08-12

Verdict: local-correction (resolved).

1 The printed exponent

In the proof of Proposition IV.2 of [CPGSV17], the right and left source-cut ranks of the k-site blocked tensor are printed as

$$r_k \ell_k = d^k.$$

The blocked physical dimension is d^k.

2 Consistent rank-product identity

Theorem III.8 states that for a simple tensor of physical dimension q, the corresponding ranks obey

$$r\ell = q^2.$$

Applying this statement to the k-site block, with $q = d^k$, gives

$$r_k \ell_k = (d^k)^2 = d^{2k}.$$

The same square occurs in the index discussion and in the later blocking argument. Thus the occurrence of d^k in the proof of Proposition IV.2 is a typographical exponent error; the consistent endpoint identity is d^{2k}.

3 Formal treatment

The simultaneous saturation theorem `MPOTensor.blockingRanks_eq_pow_mul_of_products` does not prove this rank-product identity. It takes the two endpoint identities

$$r_{k_0} \ell_{k_0} = d^{2k_0}, \qquad r_k \ell_k = d^{2k}$$

as explicit hypotheses and combines them with the independently proved blocking upper bounds. Its focused accessors are `MPOTensor.rightRank_blockTen` `sor_eq_pow_mul_of_products` and `MPOTensor.leftRank_blockTensor_eq_pow_mul_of_products`. A later application may obtain the endpoint identities from the formal counterpart of Theorem III.8 under its own assumptions.

4 Verdict

Verdict: local correction (resolved). The exponent in the proof of Proposition IV.2 is corrected from d^k to d^{2k}. This does not add an assumption beyond the rank-product identity used by that proof, but the formal saturation result keeps that identity explicit rather than claiming it from blocking alone.

References

[CPGSV17] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product unitaries: Structure, symmetries, and topological invariants. *Journal of Statistical Mechanics: Theory and Experiment*, 2017(8):083105, 2017.