

Domain Walls: Nonzero Tensors, Phases and Blocked Local Actions

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Verdict: local-correction (resolved).

Ref. [GRS24] models the excitations between two symmetry-broken ground states A_x and A_y of a matrix product unitary symmetry by domain-wall tensors e_{xy} , and describes the symmetry action on them locally: the action tensors of g on the two regions, applied to O_g acting on the chain $A_x e_{xy} A_y$, return $B_{x,y}^g$ times the chain $A_{gx} e_{gx,gy} A_{gy}$.¹ Two readings of these definitions are adopted as conventions.

1 Nonzero domain walls and phases

The source never states that a domain-wall tensor is nonzero, nor that the scalars c_{AB} , c_{BA} and $B_{x,y}^g$ are nonzero. Read literally, the zero tensor is a domain wall carried to itself with every phase, and a zero phase is allowed for a nonzero wall. Neither is intended: the tensors model excitations, and the scalars are called phases and phase factors (lines 712, 1656 and 1894). The formalization builds both conditions into the local action: a group element carries a nonzero domain wall to a nonzero image wall with a nonzero phase.² Without a nonzero wall no phase is determined, so the literal reading has no content to formalize.

2 Blocked local actions

The source draws the local action at the renormalization fixed point, with one site of A_x and one site of A_y beside the wall (lines 774–832), and states that away from the fixed point the same analysis applies after blocking (line 1358). The formalization states the local action for all words of A_x and A_y longer than a fixed buffer. The buffered form is needed even for the fixed-point example of the source: for the CZX symmetry and the product states $|0\rangle^{\otimes N}$ and $|1\rangle^{\otimes N}$, the

¹Source traceability: `Papers/2405.00439/MPU-DW.tex`, lines 643–838 (the $\mathbb{Z}_2$ case, `SingleDW`, `DWopmps`, `eq:localcdef`) and lines 1889–1933 (the finite-group case, `eq:localcdefG`, `PentLB`).

²Formal declarations: `MPOTensor.GroupFamily.BlockActionData.IsDomainWallAction`, `MPOTensor.GroupFamily.BlockActionData.IsDomainWallAction.phase_eq`.

wall $e_{BA} = -|1\rangle$ acquires its phase $c_{BA} = -1$ only with at least one site to the right of the wall.³

Verdict: resolved local fix. Domain walls are nonzero, their phases are nonzero, and the local action holds against blocked regions; with these readings the source’s relations $c_{AB}c_{BA} = L_A/L_B = \omega$ and **PentLB** are proved.

References

- [GRS24] José Garre-Rubio and Norbert Schuch. Fractional domain wall statistics in spin chains with anomalous symmetries. arXiv:2405.00439v2, 2024.

³Formal declarations: `MPOTensor.GroupFamily.BlockActionData.IsDomainWallAction`, `CZXCompression.czx_isDomainWallAction_ba`.