

One Left Shift per Site in the Periodic Group Operators

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Verdict: local-correction (resolved).

Let G be a finite group and ω a normalized three-cocycle of G with values in $U(1)$. On $\mathbb{C}[G] \otimes \mathbb{C}[G]$ write

$$W_g|k, l\rangle = \omega(g, l, l^{-1}k)|k, l\rangle, \quad L_g|k\rangle = |gk\rangle. \quad (1)$$

The periodic construction of Ref. [GRLM23], in the subsection on periodic boundary conditions, prints

$$U_g = \bigotimes_i (L_g^i \otimes L_g^{i+1}) \cdot W_g^{i,i+1} \quad (2)$$

on a ring of sites i .¹ Read literally, every site i receives one factor L_g from the pair $(i-1, i)$ and one from the pair $(i, i+1)$, so the product acts on each site by $L_g^2 = L_{g^2}$. For $G = \mathbb{Z}_2$ this would give $U_g = \prod_i CZ_{i,i+1}Z_i$ without the spin flips, contradicting the printed $\mathbb{Z}_2$ example $U_g = \prod CZ_{i,i+1}Z_i \prod X_i$ on line 2224.

The pair $L_g^i \otimes L_g^{i+1}$ descends from the arbitrary-boundary operator $O_g = \bigotimes_i (L_g^{[i]_l} \otimes L_g^{[i]_r}) \cdot W_g^{[i]_l, [i]_r} \cdot \mathcal{P}^{[i]_r, [i+1]_l}$ of line 2094, where the two factors act on the two halves of one site. The isometry $\Gamma : |g, g\rangle_{[i]_r, [i+1]_l} \mapsto |g\rangle_i$ merges the halves $[i]_r$ and $[i+1]_l$ into one site, on which the two shifts act jointly as a single L_g . The tensor $\hat{T}_g$ drawn on lines 2208–2222 carries exactly one L_g per site. The intended operator is therefore

$$U_g = \left(\bigotimes_i L_g^i \right) \cdot \prod_i W_g^{i,i+1}, \quad (3)$$

with entries $\langle s|U_g|t\rangle = \prod_i \delta_{s_i, gt_i} \omega(g, t_{i+1}, t_{i+1}^{-1}t_i)$. With this reading $U_g U_h = U_{gh}$ follows from the cocycle identity, whose correction factors $\omega(g, h, t_{i+1})/\omega(g, h, t_i)$ telescope around the ring, and $U_e = \text{id}$ from normalization.²

¹Source traceability: `Papers/2203.12563/REsubmission.tex`, lines 2068, 2070 and 2204–2222.

²Formal declarations: `MPOTensor.GroupCocycle.mpo_tensor_apply_eq_leftShift_wGate`, `MPOTensor.GroupCocycle.mpo_tensor_mul`, `MPOTensor.GroupCocycle.mpo_tensor_one` and `MPOTensor.GroupCocycle.mpo_tensor_cyclicTwo` in `TNLean/MPS/MPU/GroupCocycleMPO` and `TNLean/MPS/MPU/GroupCocycleMPO/Instances`.

Verdict: resolved local fix. The printed tensor and the printed $\mathbb{Z}_2$ example both fix the reading (3).

References

- [GRLM23] José Garre-Rubio, Laurens Lootens, and András Molnár. Classifying phases protected by matrix product operator symmetries using matrix product states. *Quantum*, 7:927, 2023. Source coordinates in this repository use arXiv:2203.12563v3.