

# The Final Index in the Block-Dependent Two-Cochain Coboundary

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**Verdict:** local-correction (resolved).

## 1 Printed coboundary

For a block-dependent scalar function  $f_{g,h}^x$ , Ref. [FRBC25, lines 5918–5921] prints

$$[df]_{g,h,k}^x = \frac{f_{g,hk}^x f_{h,k}^x}{f_{g,h}^{kx} f_{gh,x}^x}. \quad (1)$$

The final subscript  $x$  is a block label, whereas that position is a group index. The printed denominator is therefore ill-typed in the notation of the source.

## 2 Corrected index

The final group index must be  $k$ :

$$[df]_{g,h,k}^x = \frac{f_{g,hk}^x f_{h,k}^x}{f_{g,h}^{kx} f_{gh,k}^x}. \quad (2)$$

Typing rules out the printed block label  $x$ , but does not by itself distinguish among the group elements  $g$ ,  $h$ , and  $k$ . The standard generalized coboundary cancellation  $d(df) = 1$  determines the final index as  $k$ ; this is also the identity used in the ensuing generalized-cocycle argument of Lemma `lemma:G_cocycle`.

## 3 Formal treatment

The formal two-cochain coboundary uses Eq. 2, and its next coboundary is proved to be one. No hypothesis or conclusion is changed.<sup>1</sup>

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<sup>1</sup>Formal declarations: `TNLean.Algebra.LSymbol.coboundary` and `TNLean.Algebra.GeneralizedThreeCochain.coboundary_coboundary` in `TNLean/Algebra/GeneralizedCocycle`.

**Verdict: resolved local fix.** The source’s declared argument types rule out the printed block label  $x$ . The coboundary cancellation  $d(df) = 1$ , together with its subsequent generalized-cocycle use, determines the final group index as  $k$ .

## References

- [FRBC25] Adrián Franco Rubio, Arkadiusz Bochniak, and J. Ignacio Cirac. Symmetry defects and gauging for quantum states with matrix product unitary symmetries, 2025.