

The Repeated Case in the Scalar Three-Cocycle Normalization

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Verdict: local-correction (resolved).

1 Setting

Let G be a group with identity element e , and write $\mathbb{C}^\times$ for the multiplicative group of nonzero complex numbers. A scalar 3-cochain is a function $\omega: G^3 \rightarrow \mathbb{C}^\times$, written $\omega(g, h, k)$; in Ref. [FRBC25] it is the anomaly 3-cocycle attached to the fusion tensors of a matrix product unitary representation of G . The question at issue is which values of ω the paper's standing gauge conventions set to 1.

2 Printed normalization

In its list of standing gauge conventions, Ref. [FRBC25] states that every fusion tensor involving e is trivial and that ω is consequently normalized. The accompanying display prints

$$\omega(g, h, e) = \omega(g, h, e) = \omega(g, h, e) = 1, \quad \text{for every } g, h. \quad (1)$$

Thus the same identity-axis case appears three times, while the other two identity axes do not appear.

3 Corrected convention

The preceding sentence requires triviality whenever any fusion input is the identity. The normalized scalar 3-cochain convention is therefore

$$\omega(e, h, k) = \omega(g, e, k) = \omega(g, h, e) = 1, \quad \text{for every } g, h, k. \quad (2)$$

Equation 2 replaces the three repeated copies in Eq. 1 by the three possible identity-axis cases. It changes neither the scalar codomain nor the 3-cocycle equation.

4 Formal treatment

The formal definition uses Eq. 2. This is a local correction of the repeated case in the printed display, not an additional hypothesis on scalar 3-cochains.¹

Verdict: resolved local fix. The source’s surrounding standing convention determines all three identity-axis conditions, so the repetition is typographical.

References

- [FRBC25] Adrián Franco Rubio, Arkadiusz Bochniak, and J. Ignacio Cirac. Symmetry defects and gauging for quantum states with matrix product unitary symmetries, 2025.

¹Formal declaration: `TNLean.Algebra.ScalarThreeCochain.IsNormalized` in `TNLean/Algebra/ScalarThreeCocycle`.