

The Representative Index in the First Stabilizer Factor

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Verdict: local-correction (resolved).

Let a group G act transitively on X . Fix $x_0 \in X$ and choose $k_x \in G$ such that $k_x x_0 = x$. For $g \in G$ and $x \in X$, define

$$t_k(g, x) = k_{gx}^{-1} g k_x. \quad (1)$$

Then $t_k(g, x)x_0 = x_0$, so $t_k(g, x)$ lies in $H = \text{Stab}_G(x_0)$, and direct cancellation gives

$$t_k(g_1 g_2, x) = t_k(g_1, g_2 x) t_k(g_2, x). \quad (2)$$

The displayed definition of the two stabilizer factors in the lemma preceding the block-independence proposition of Ref. [FRBC25] prints $h_1 = k_{g_1 g_2 x}^{-1} g_1 k_{g_1 x}$. This expression does not generally stabilize x_0 . Resolving the g_2 action first in (2) gives the corrected factors

$$h_2 = k_{g_2 x}^{-1} g_2 k_x = t_k(g_2, x), \quad (3)$$

$$h_1 = k_{g_1 g_2 x}^{-1} g_1 k_{g_2 x} = t_k(g_1, g_2 x). \quad (4)$$

Thus the final representative in h_1 is $k_{g_2 x}$, not $k_{g_1 x}$. No normality or finiteness assumption is needed.¹

Verdict: resolved local fix. The chosen-representative product determines the corrected index uniquely.

References

- [FRBC25] Adrián Franco Rubio, Arkadiusz Bochniak, and J. Ignacio Cirac. Symmetry defects and gauging for quantum states with matrix product unitary symmetries, 2025.

¹Formal declarations: `TNLean.Algebra.StabilizerRepresentatives.transitionElement`, `TNLean.Algebra.StabilizerRepresentatives.coe_transitionElement`, and `TNLean.Algebra.StabilizerRepresentatives.transitionElement_mul` in `TNLean/Algebra/StabilizerTransition`.