

The Positive Primitive in the Generalized-Cocycle Corollary

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Verdict: local-correction (resolved).

1 Source argument

Let G be finite and let the positive generalized 2-cocycle Ω take values in the embedded subgroup $\mathbb{R}_{>0} \subset \mathbb{C}^\times$. The proof of [FRBC25, Corollary `cor:positive_trivial`, lines 5950–5960] invokes the preceding identity

$$\Omega^{|G|} = d\delta, \quad \delta_g^x = \det X_g^x, \quad (1)$$

and then takes a positive $|G|$ -th root.

The determinant cochain δ need not itself be positive. Indeed, the regular-action matrix factors as a permutation matrix times a diagonal matrix, and the permutation determinant can equal -1 . Thus a positive root of δ_g^x is not defined in general.

2 Positive primitive

Take the pointwise complex norm before taking the root:

$$\chi_{0,g}^x = |\delta_g^x|^{1/|G|} = |\det X_g^x|^{1/|G|}. \quad (2)$$

Since $\delta_g^x \neq 0$, every value of χ_0 lies in $\mathbb{R}_{>0} \subset \mathbb{C}^\times$. Multiplicativity of the complex norm and positivity of Ω turn Eq. 1 into

$$\Omega^{|G|} = d|\delta|. \quad (3)$$

Positive real powers are multiplicative, so $(d\chi_0)^{|G|} = \Omega^{|G|}$. Injectivity of the $|G|$ -th power on $\mathbb{R}_{>0}$ gives $d\chi_0 = \Omega$.

3 Formal treatment

The formal construction takes the pointwise norm of the determinant cochain and then its positive $|G|$ -th root. The block label x is retained in Eq. 2. This changes no hypothesis or conclusion of the corollary.¹

Verdict: resolved local correction. The norm removes the possible permutation sign while preserving the coboundary identity required by the positive-root argument.

References

- [FRBC25] Adrián Franco Rubio, Arkadiusz Bochniak, and J. Ignacio Cirac. Symmetry defects and gauging for quantum states with matrix product unitary symmetries, 2025.

¹Formal declarations: `TNLean.Algebra.LSymbol.positivePrimitive` and `TNLean.Algebra.LSymbol.exists_positive_coboundary` in `TNLean/Algebra/PositiveGeneralizedCocycle`.