

Source Boundary for MPU Action Tensors and Stabilizer Data

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Verdict: open-gap (open).

1 Scope and conclusion

The target-paper coordinates in this note refer to `Papers/2502.20257/main.tex`. The external coordinates refer to the following official arXiv sources and their named L^AT_EX files.

- Molnár–Ge–Schuch–Cirac, *A generalization of the injectivity condition for Projected Entangled Pair States*, arXiv:1706.07329v2, `cornerproblem.tex`, under the arXiv non-exclusive distribution license 1.0.
- Garre-Rubio–Lootens–Molnár, *Classifying phases protected by matrix product operator symmetries using matrix product states*, arXiv:2203.12563v3, `REsubmission.tex`, under CC BY 4.0.
- Garre-Rubio–Schuch, *Fractional domain wall statistics in spin chains with anomalous symmetries*, arXiv:2405.00439v2, `MPU-DW.tex`, under CC BY 4.0.

The line ranges below are citations into these named source files rather than coordinates reconstructed from reformatted PDFs.

The discussion of an MPU action on MPSs in Ref. [FRBC25, lines 1782–1931] uses two logically different results. The existence and scalar uniqueness of the action tensors come from the MPS reduction theorem of Ref. [MGSC18]. The description of scalar L -symbols by stabilizer data comes from Sections 3 and 4.1 of Ref. [GRLM23]. The second paper also proves an existence theorem for action tensors, but under an arbitrary-boundary invariance hypothesis that is stronger than the periodic-boundary hypothesis in the first paper and in Ref. [FRBC25].

Thus there are two valid existence routes, but they cannot be interchanged. For the simple-MPU setting of Ref. [FRBC25], the source-faithful route is the positive-length MPS reduction theorem, followed by its nilpotency and scalar-uniqueness results. Appendix A of Ref. [GRLM23] applies only after arbitrary-boundary invariance has been supplied. Once action and fusion tensors exist,

the scalar analysis of Section 4.1 of that paper applies independently of which existence route produced them.

This distinction is the present open formal boundary. The scalar L -symbol relations are already formalized, whereas the tensor-valued reduction and action data are not.

2 The reduction route to action tensors

Let A_x be an injective MPS tensor for every x in a set X , and let $B_{g,x}$ denote the MPS tensor obtained by stacking the MPU tensor $\mathcal{U}_g$ on A_x and vectorizing the physical legs. The periodic symmetry assumption is equality of the positive-length MPS vectors:

$$\mathcal{V}_N(B_{g,x}) = \mathcal{V}_N(A_{g \cdot x}), \quad N \geq 1. \quad (1)$$

Here the dot denotes the left action fixed below. The target tensor $A_{g \cdot x}$ is injective and therefore normal.

Proposition 20 of Ref. [MGSC18, source lines 3124–3133, with proof at lines 3815–3939] applies to a normal target tensor A and an arbitrary tensor B . Its printed hypothesis allows an exponential scalar λ^N , but its unscaled conclusion is false when $\lambda \neq 1$. The source proof establishes the equality specialization; the corrected nonzero- λ conclusion retains the factor λ^n on words of length n . This source defect and its scalar counterexamples are recorded in the dedicated paper-gap note. In the exact symmetry relation (1), $\lambda = 1$, so the application here is unaffected. The equality specialization supplies matrices $V_{g,x}$ and $W_{g,x}$ such that

$$V_{g,x} W_{g,x} = \mathbf{1}, \quad (2)$$

$$V_{g,x} B_{g,x}^{i_1} \cdots B_{g,x}^{i_n} W_{g,x} = A_{g \cdot x}^{i_1} \cdots A_{g \cdot x}^{i_n}, \quad n \geq 1. \quad (3)$$

The proof obtains (2) by taking the empty interior word in the local reduction identity; it does not require equality of the two length-zero periodic MPS coefficients.

The two matrices in (3) are the source of the action tensors $A_{g,x}^\Gamma$ and $A_{g,x}^\Gamma$ in the target paper, after matching the left and right virtual-leg orientations in its diagrams. Proposition 21 and Definition 8 of Ref. [MGSC18, source lines 3142–3152, with proof at lines 3943–3986] introduce the residual matrices

$$R_{g,x}^i = B_{g,x}^i - W_{g,x} A_{g \cdot x}^i V_{g,x} \quad (4)$$

and prove that their generated algebra is nilpotent. The nilpotency length $\ell_{g,x}$ is the least length after which every residual word vanishes:

$$R_{g,x}^{i_1} \cdots R_{g,x}^{i_n} = 0, \quad n \geq \ell_{g,x}. \quad (5)$$

Nilpotency alone does not remove the mixed terms in an expansion of B . The additional input is the intended residual-word expansion of Appendix B and

its preceding sandwiched-residual lemma [MGSC18, source lines 3946–3970 and 3993–4005]. Suppress g, x temporarily, write an empty product as the identity, and let $\mathbf{i} = (i_1, \dots, i_n)$. The sandwiched residual words obey

$$VR^{i_1} \dots R^{i_r} W = 0, \quad r > 0. \quad (6)$$

The first equality printed in Lemma `B_expand`, at source lines 3993–4005, is false as printed and admits the following local correction. Its weak inequality $0 \leq r \leq s \leq n$ includes terms with an empty A -word and omits the pure residual word. At $n = 1$ the printed right-hand side is $WVR^i + WA^iV + R^iWV$, rather than $R^i + WA^iV$. A nondegenerate reduction counterexample and the correction are recorded in the dedicated false-source note [con26a].

Expanding $B^i = R^i + WA^iV$ and using (6) shows that the correct full expansion is

$$B^{i_1} \dots B^{i_n} = R^{i_1} \dots R^{i_n} + \sum_{0 \leq r < s \leq n} R^{i_1} \dots R^{i_r} WA^{i_{r+1}} \dots A^{i_s} VR^{i_{s+1}} \dots R^{i_n}. \quad (7)$$

Here the strict inequality $r < s$ says that the central A -word is nonempty. Contracting this corrected identity and using residual nilpotency gives the other two equalities printed in Lemma `B_expand`:

$$VB^{i_1} \dots B^{i_n} = \sum_{\max(0, n-\ell) \leq s \leq n} A^{i_1} \dots A^{i_s} VR^{i_{s+1}} \dots R^{i_n}, \quad (8)$$

$$B^{i_1} \dots B^{i_n} W = \sum_{0 \leq r \leq \min(\ell, n)} R^{i_1} \dots R^{i_r} WA^{i_{r+1}} \dots A^{i_n}. \quad (9)$$

For $n < \ell$, the endpoints $s = 0$ and $r = n$ in these contracted sums are the pure residual terms $VR^{\mathbf{i}}$ and $R^{\mathbf{i}}W$. For $n \geq \ell$, the displayed endpoints with residual length exactly ℓ are zero by the source definition of nilpotency length. They are retained to match the printed bounds; the strictly surviving ranges are $n - \ell < s \leq n$ and $0 \leq r < \ell$.

Now let $\mathbf{p}$ and $\mathbf{q}$ both have length $m \geq \ell$. The pure residual term in the corrected full expansion of $B^{\mathbf{p}\mathbf{c}\mathbf{q}}$ vanishes. A central A -word which starts after $\mathbf{p}$ leaves a residual prefix of length at least ℓ , and one which ends before $\mathbf{q}$ leaves a residual suffix of length at least ℓ . Hence every surviving central A -word crosses all of $\mathbf{c}$. The two contracted equations enumerate precisely these surviving terms and give

$$B^{\mathbf{p}}WA^{\mathbf{c}}VB^{\mathbf{q}} = B^{\mathbf{p}\mathbf{c}\mathbf{q}}, \quad |\mathbf{p}| = |\mathbf{q}| = m \geq \ell. \quad (10)$$

This is the algebraic content of the exterior action identity (59); the interior identity (60) is the reduction relation (3) and permits an interior length $n \geq 0$ [FRBC25, lines 1787–1870].

Neither one-sided identity $VB^{\mathbf{i}} = A^{\mathbf{i}}V$ nor $B^{\mathbf{i}}W = WA^{\mathbf{i}}$ follows. For example, $A^1 = 1$, $W = (1, 0)^T$, $V = (1, 0)$, and $B^1 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ satisfy the retraction and all-word reduction relations, while $R^1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ has $(R^1)^2 = 0$ but

$V(B^1)^n = (1, 1) \neq V$ for every $n > 0$. The full mixed-term expansions, rather than residual nilpotency by itself, are therefore load-bearing for (10).

Theorem 22 of Ref. [MGSC18, source lines 3154–3162, with proof at lines 3990–4035] gives the precise uniqueness statement. If (V, W) and $(\widetilde{V}, \widetilde{W})$ are two reductions whose nilpotency lengths are at most N_0 , then there is $c \in \mathbb{C}^\times$ such that, for every word of length $n > 2N_0$,

$$VB^{i_1} \dots B^{i_n} = c\widetilde{V}B^{i_1} \dots B^{i_n}, \quad (11)$$

$$B^{i_1} \dots B^{i_n}W = c^{-1}B^{i_1} \dots B^{i_n}\widetilde{W}. \quad (12)$$

This is tailwise scalar uniqueness. It is not the unconditional assertion $V = c\widetilde{V}$ and $W = c^{-1}\widetilde{W}$ as bare matrices.

Theorem 1 of Ref. [GRS24, source lines 358–377] restates the nilpotency-one case in the form used for MPU products and MPU actions. Its hypotheses give

$$VB^i\widehat{V} = A^i, \quad (13)$$

$$B^i\widehat{V}A^jVB^k = B^iB^jB^k, \quad (14)$$

$$V\widehat{V} = \mathbf{1}. \quad (15)$$

The statement is explicitly described there as informal: its footnote assumes that the off-diagonal residual has nilpotency length one and says that this is achieved by blocking. Likewise, the target paper first permits $m \geq \ell$ and then assumes that all nilpotency lengths have been reduced to one by blocking [FRBC25, lines 1870–1874]. Consequently a formal action-tensor theorem must either retain ℓ in its identities or prove the blocking step before using the one-site identities. Simplicity of the MPU is not, by itself, a cited proof that an unblocked stacked tensor has $\ell = 1$.

The relevant global equality in the formal library is positive-length equality for possibly different bond dimensions. Full equality at length zero would compare the traces of two identity matrices and therefore force equality of the bond dimensions, although the stacked tensor $B_{g,x}$ and the reduced tensor $A_{g \cdot x}$ need not have equal bond dimensions. Thus the correct formal input is `MPSTensor.SameMPVPos`, not `MPSTensor.SameMPV`.

3 The arbitrary-boundary route is different

Section 2 of Ref. [GRLM23, source lines 302–331] defines an MPS arbitrary-boundary subspace by allowing every virtual boundary matrix X , and an MPO algebra by allowing every MPO boundary matrix C . Its closedness condition requires one boundary matrix, independent of the system size, to represent each product of arbitrary-boundary MPOs. Section 3 then defines symmetry by the stronger condition that for every C and X there is a boundary matrix $Y(C, X)$, again independent of N , such that

$$O_{T,C}^N |\psi_{A,X}^N\rangle = |\psi_{A,Y(C,X)}^N\rangle, \quad \text{for every } C, X, N. \quad (16)$$

This is Equation (7) of that source [GRLM23, source lines 429–460]. Its Appendix A starts from the size-independent linear boundary map, takes a minimal-rank decomposition, and uses the one-site and two-site identities plus block injectivity to obtain fusion tensors and their orthogonality [GRLM23, source lines 2305–2454]. The final sentence applies the same construction to (16) to obtain action tensors and their orthogonality [GRLM23, source line 2455].

By contrast, the simple-MPU hypothesis together with the independent periodic symmetry assumption gives (1) for the distinguished closed boundaries. Simplicity alone does not assert this equality. These hypotheses do not quantify over C and X , and they do not supply the boundary map used in Appendix A. The mismatch is structural, not merely a missing lemma. Ref. [GRLM23, source lines 1211–1216] proves that an injective group MPO satisfying $U_g U_h = U_{gh}$ and $U_e = \mathbf{1}$ must be on-site, with trivial 3-cocycle, if its fusion tensors also satisfy the arbitrary-boundary closedness condition. Hence an anomalous injective MPU representation with $U_e = \mathbf{1}$ cannot satisfy that condition.

There is a second difference. In the arbitrary-boundary group algebra of Ref. [GRLM23, source lines 638–683], the neutral operator O_e is generally a projector onto a relevant periodic-boundary subspace, not the identity on the full Hilbert space, and the operators need not be unitary. The target paper instead uses simple injective MPUs, each unitary on the full periodic chain, with $\mathcal{U}_e = \mathbf{1}$. The reduction theorem of the preceding section is designed for this latter situation.

4 Left-action and scalar conventions

The target paper uses a left action: $g \cdot x$ is the block obtained by applying g to x , and $gh \cdot x$ means that h acts first and then g . Its transitivity assumption is that for every $x, y \in \mathbf{X}$ there is $g \in G$ with $y = g \cdot x$ [FRBC25, lines 1782–1785]. With this convention, the scalar compatibility equation is

$$L_{g,hk}^x L_{h,k}^x = \omega(g, h, k) L_{g,h}^{k \cdot x} L_{gh,k}^x. \quad (17)$$

The shifted block is $k \cdot x$, not $g \cdot x$ or $h \cdot x$, because the comparison begins with the rightmost action. Equation (17) is Equation (64) of the target paper [FRBC25, lines 1920–1924] and agrees with the group specialization in Ref. [GRLM23, source lines 683–719].

The target paper scales its reflected- Γ action tensor by $\gamma_{g,x}$ and its Γ action tensor by $\gamma_{g,x}^{-1}$:

$$A_{g,x}^\Gamma \mapsto \gamma_{g,x} A_{g,x}^\Gamma, \quad A_{g,x}^\Gamma \mapsto \gamma_{g,x}^{-1} A_{g,x}^\Gamma. \quad (18)$$

Together with its fusion-tensor convention this gives

$$L_{g,h}^x \mapsto \frac{\gamma_{gh,x} \beta_{g,h}}{\gamma_{g,h \cdot x} \gamma_{h,x}} L_{g,h}^x. \quad (19)$$

These are Equations (61) and (63) of Ref. [FRBC25, lines 1870–1919]. Equation (19) of Ref. [GRLM23, source lines 720–726] prints the reciprocal factor. The two formulas use inverse names for both scalar gauges: replacing the 2023 parameters by γ^{-1} and β^{-1} gives (19). The compatibility equation is unchanged. The formal library follows the target convention, so a proof may not import the 2023 gauge formula without this inversion.

5 Stabilizer data and the meaning of (H, ψ)

Assume now that G is finite, $\mathbf{X}$ is nonempty, and the left action is transitive. After choosing $x_0 \in \mathbf{X}$, the subgroup in Section 4.1 of Ref. [GRLM23, source lines 731–749] is precisely the stabilizer

$$H = \text{Stab}_G(x_0) = \{h \in G \mid h \cdot x_0 = x_0\}. \quad (20)$$

It is not an additional “largest” subgroup and it need not be normal. Transitivity gives an equivariant bijection of sets

$$\begin{aligned} G/H &\longrightarrow \mathbf{X}, & gH &\longmapsto g \cdot x_0, \\ |\mathbf{X}| &= |G/H|. \end{aligned} \quad (21)$$

Here G/H is the left-coset set. It is not a quotient group unless H is normal. Replacing x_0 by $g \cdot x_0$ replaces H by the conjugate subgroup gHg^{-1} , so the basepoint-free datum is a conjugacy class of stabilizer data, not a distinguished subgroup.

Restricting (17) to H shows that $\omega|_H$ is cohomologically trivial. In the convention printed in Ref. [GRLM23, source lines 740–749], one chooses a 2-cochain β satisfying

$$\omega(h_1, h_2, h_3) \frac{\beta_{h_1, h_2} \beta_{h_1 h_2, h_3}}{\beta_{h_2, h_3} \beta_{h_1, h_2 h_3}} = 1, \quad h_1, h_2, h_3 \in H, \quad (23)$$

and then defines the representative

$$\psi_{x_0}(h_1, h_2) = \frac{L_{h_1, h_2}^{x_0}}{\beta_{h_1, h_2}}. \quad (24)$$

In that chosen trivializing gauge, ψ_{x_0} is a 2-cocycle. Changing the action-tensor gauge changes it by a 2-coboundary, so the invariant is its class $[\psi_{x_0}] \in H^2(H, \mathbb{C}^\times)$. Before choosing a trivializer for $\omega|_H$, the corresponding object is a trivializing 2-cochain rather than a canonically based element of H^2 .

Equation (20) of Ref. [GRLM23] makes the representative-level choices explicit; it is at lines 752–759 of the official v3 source. The prose at line 754 announces a solution for L_{g_2, g_1}^α , whereas the displayed equation at lines 755–758 is for L_{g_1, g_2}^α and first resolves the g_2 action, then the g_1 action. The prose has the swapped-order typo; the display fixes the convention recorded here. Choose

a representative $k_\alpha \in G$ for every coset and define $h_2, h_1 \in H$ by $g_2 k_\alpha = k_\beta h_2$ and $g_1 k_\beta = k_\gamma h_1$. The source gives

$$L_{g_1, g_2}^\alpha = \frac{\omega(g_1 g_2 k_\alpha, h_2^{-1}, h_1^{-1}) \psi(h_1, h_2)}{\omega(g_1, g_2 k_\alpha, h_2^{-1}) \omega(g_1, g_2, k_\alpha)}. \quad (25)$$

Thus Equation (20) constructs a chosen scalar L -symbol representative from a chosen representative of ω , a chosen trivialization and cocycle representative, a basepoint, and coset representatives. It is not a canonical formula on cohomology classes, and it does not construct action tensors or an MPU realization. The formula is compatible exactly as printed only in a fusion gauge in which ω is one on $H \times H \times H$; for a general ω with $\omega|_H = d\beta$ it can fail, and the correction is recorded in [con26b].

For fixed (G, ω, H) , Ref. [GRLM23, source line 761] states that gauge classes of solutions form a torsor for $H^2(H, \mathbb{C}^\times)$. A choice of trivialization identifies that torsor with the group, but in general there is no canonical solution corresponding to the identity cohomology class. Accordingly, the notation (H, ψ) has three levels which should not be conflated:

1. in Equation (20), H and ψ are chosen representatives, together with the unprinted auxiliary gauge choices just listed;
2. for scalar classification, the invariant is stabilizer data up to basepoint conjugacy and a gauge class represented by ψ ;
3. for phases, the invariant is the resulting equivalence class of L -symbols, under the hypotheses of the phase theorem.

The phase theorem itself concerns periodic Hamiltonians with exact MPS ground spaces, equivalent F -symbols, and the restricted symmetric gapped paths specified in that paper [GRLM23, source lines 1219–1272, especially the main result at line 1231]. It is not a classification of arbitrary Hamiltonian phases, raw cocycle representatives, action tensors, or MPU tensor data. Conversely, the scalar construction (25) proves existence of an L -symbol solution for admissible chosen data, but not existence of tensor data realizing every such solution in the simple-MPU setting.

6 Present formal boundary

The existing library separates the following layers.

- `TNLean` defines `MPSTensor.IsPrimitivePaper`, the unnormalized uniform vector-spreading formulation of paper primitivity, and `MPSTensor.HasEventuallyFullKrausRank`. The latter is equivalent to `QICLean's Kraus.IsNormal` by `MPSTensor.hasEventuallyFullKrausRank_iff_isNormal`. The reverse bridge from `MPSTensor.IsPrimitivePaper` to the word-span predicate in `MPSTensor.primitivePaper_iff_hasEven`

`tuuallyFullKrausRank` assumes left-canonical normalization. Thus an exact named formal predicate for source primitivity already exists, while any passage from it to word-span normality must retain its left-canonical normalization hypothesis.

QICLean also defines `Kraus.IsInjective` and `Kraus.IsNBlkInjective` for one matrix family. The latter says that the length- N words of one tensor span its full matrix algebra. None of these predicates is the paper’s direct-sum block-injective condition

$$A^i = \bigoplus_{x \in X} A_x^i, \quad \text{span}\{A_x^i\}_i = \text{Mat}_{D_x}(\mathbb{C}) \quad \text{for every } x. \quad (26)$$

The direct-sum assembly is available through `MPSTensor.toTensorFromBlocks` and the richer `MPSTensor.SectorDecomposition` infrastructure, but no existing predicate attaches a transitive group action and an MPU action to that family.

- `TNLean` defines `MPOTensor.IsMPU` by unitarity for every system size $N > 1$, following the 2017 MPU convention, and defines `MPOTensor.IsMPUSimple` by the two local simple-tensor contractions. It also has the product tensor `MPOTensor.multTensor` and the doubled-index view `MPOTensor.toMPSTensor`. It has no group-indexed MPU representation object and no tensor for the stacked action of an MPU on an MPS. Those notions must precede the reduction theorem; they should not be inferred from the scalar L -symbol API.
- `MPSTensor.SameMPVPos` expresses the global equality in (1) across different bond dimensions. The missing result is the Molnár–Ge–Schuch–Cirac reduction package: (2)–(5), its transport under blocking, and the tailwise uniqueness (11)–(12).
- `Mathlib’s MulAction`, `MulAction.IsPretransitive`, `MulAction.stabilizer`, and `MulAction.orbitEquivQuotientStabilizer` provide the correct action, transitivity, stabilizer, and coset-set language for (20)–(21). Transitivity is represented by pretransitivity together with nonemptiness. No normality hypothesis on H should be introduced.
- `TNLean’s TNLean.Algebra.LSymbol` is the scalar function $X \rightarrow G \rightarrow G \rightarrow \mathbb{C}^\times$. `TNLean.Algebra.LSymbol.IsCompatible` is exactly (17), and `TNLean.Algebra.LSymbol.gauge` is exactly (19). The name `TNLean.Algebra.ActionTensorGauge` denotes only the scalar cochain $\gamma_{g,x}$; it is not tensor data. These declarations deliberately assert neither existence of action tensors nor their attachment to an MPU.
- `TNLean.Algebra.H2` is the quotient of genuine $\mathbb{C}^\times$ -valued 2-cocycles by coboundary equivalence. Its coefficient type is `Units ℂ`; despite prose in some module comments using $U(1)$, the implemented coefficient group

matches the $\mathbb{C}^\times$ notation of the cited papers. The representative reconstruction (25) in a fusion gauge trivial on the stabilizer, the classification of scalar solutions up to action-tensor gauge, and the torsor of solution classes (the group $H^2(H, \mathbb{C}^\times)$ under pointwise multiplication of cocycles, acting freely and transitively on action-gauge classes) are formalized in `TNLean/Algebra/StabilizerCocycleReconstruction`.

- The on-site, single-block specialization is already represented by `MPSTensor.IsOnSiteSymmetric` and `MPSTensor.virtual_rep_of_symmetric_injective`. It should be reused when $H = G$ and the anomaly is trivial, but it is not a substitute for an anomalous MPU acting on several injective blocks. Likewise, the MPDO structure named `MPOTensor.BNTFusionTensorClause` concerns the CPSV16 renormalization-fixed-point fusion clause and is not the group-MPU fusion tensor of the present sources.

The tensor-valued reduction package, rather than a new cohomology interface, is therefore the next mathematical prerequisite. Until it exists, the formal scalar L -symbol results must not be described as proving the existence or classification of MPU action tensors.

7 Verdict

The cited action-tensor claim is supported by Ref. [MGSC18], provided that one retains the normal target, positive-length periodic equality, residual nilpotency, and finite blocking used by that theorem. Its printed full residual expansion requires the pure-residual and nonempty-middle correction in (7). Its scalar uniqueness is the tailwise statement (11)–(12). The arbitrary-boundary theorem of Ref. [GRLM23] is not an alternative proof under the simple-MPU hypotheses.

The stabilizer classification is a classification of gauge classes of scalar L -symbol solutions, and under the separate phase hypotheses it classifies the corresponding exact-MPS symmetric phases. The notation (H, ψ) does not by itself denote tensor data, and the solution set is naturally an $H^2(H, \mathbb{C}^\times)$ -torsor. The formal gap remains open exactly at the reduction-to-action-tensor layer; no new tensor data or cohomology interface is introduced in this note.

References

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