

The Repeated Determinant Factor in the Generalized-Cocycle Lemma

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Verdict: local-correction (resolved).

1 Setting

Let G be a finite group acting on a set X of block labels, and let $\Omega: X \times G^2 \rightarrow \mathbb{C}^\times$ satisfy the generalized cocycle equation $d\Omega = 1$. Ref. [FRBC25, Lemma `lemma:G_cocycle`] regards the group algebra $\mathbb{C}G$ as a Hilbert space of dimension $|G|$, defines the regular-action operators X_g^x on it by $X_g^x|k\rangle = \Omega_{g,k}^{k^{-1}x}|gk\rangle$, and uses the generalized coboundary $[d\chi]_{g,h}^x = \chi_g^{hx}\chi_h^x/\chi_{gh}^x$ of a block-dependent one-cochain χ .

2 Printed determinant identity

In the proof of [FRBC25, Lemma `lemma:G_cocycle`], the regular-action matrices satisfy

$$X_g^{hx} X_h^x = \Omega_{g,h}^x X_{gh}^x. \quad (1)$$

The next display prints the numerator $\det(X_g^{hx}) \det(X_g^x)$, repeating the index g in the second factor.

3 Corrected determinant identity

Take determinants on both sides of Eq. 1. On the left the determinant is multiplicative; on the right the scalar $\Omega_{g,h}^x$ multiplies an operator on the $|G|$ -dimensional space $\mathbb{C}G$, so it leaves the determinant scaled by its $|G|$ -th power. Before any division this gives

$$\det(X_g^{hx}) \det(X_h^x) = (\Omega_{g,h}^x)^{|G|} \det(X_{gh}^x). \quad (2)$$

The exponent $|G|$ in the conclusion of the lemma therefore originates in the scalar-determinant power law, applied to the dimension of $\mathbb{C}G$. Each X_g^x factors

as a regular permutation operator times an invertible diagonal operator, so $\det(X_{gh}^x) \neq 0$, and dividing Eq. 2 by it gives

$$(\Omega_{g,h}^x)^{|G|} = \frac{\det(X_g^{hx}) \det(X_h^x)}{\det(X_{gh}^x)}. \quad (3)$$

Thus the second numerator factor carries the index h . This is also the factor required by the stated choice $\chi_g^x = \det(X_g^x)$ and the generalized coboundary $[d\chi]_{g,h}^x = \chi_g^{hx} \chi_h^x / \chi_{gh}^x$.

4 Formal treatment

The formal proof takes determinants of Eq. 1 and uses Eq. 3. This changes no hypothesis or conclusion of the lemma.¹

Verdict: resolved local fix. The operator identity, generalized-coboundary definition, and declared cochain determine the missing index uniquely.

References

- [FRBC25] Adrián Franco Rubio, Arkadiusz Bochniak, and J. Ignacio Cirac. Symmetry defects and gauging for quantum states with matrix product unitary symmetries, 2025.

¹Formal declaration: `TNLean.Algebra.LSymbol.pow_card_eq_coboundary` in `TNLean/Algebra/GeneralizedCocycle`.