

Wraparound in the Orbit-Phase Telescoping Product

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Verdict: local-correction (resolved).

1 The printed step

In the proof of Theorem 4.1 of [DICCSPG17], at `Papers/1708.00029/main.tex:801--804`, every residue u modulo m is written as $u = \alpha_u + pk_u$ with $0 \leq \alpha_u < r = \gcd(m, p)$ and $0 \leq k_u < q = m/r$. The phases c_α , $0 \leq \alpha < r$, are given in the source construction together with the hypothesis $c_\alpha^q = 1$ ([DICCSPG17], `Papers/1708.00029/main.tex:784`, where they are written $c_{j,\alpha}$ with $c_{j,\alpha}^{m_j/r_j} = 1$); the note does not derive this condition. The local phases $d_u = f_{u+1}/f_u$ with $f_u = c_{\alpha_u}^{k_u}$ are claimed to satisfy

$$\prod_{k=0}^{p-1} d_{u+k} = \frac{c_{\alpha_{u+p}}^{k_{u+p}}}{c_{\alpha_u}^{k_u}} = c_{\alpha_u} \quad (1)$$

“since by definition $k_{u+p} = k_u + 1$ ”. For the bounded representative $k_u = q - 1$ this is false: then $k_{u+p} = 0$.

2 Correction

Adding p fixes α_u and replaces k_u by $k_u + 1$ modulo q . Since $c_\alpha^q = 1$, the power $c_{\alpha_u}^{k_{u+p}}$ equals $c_{\alpha_u}^{k_u+1}$ in every case, including $k_u = q - 1$, so (1) holds as stated. The correction concerns only the justification; the identity and the theorem are unchanged.

3 Formal statement

The coordinate shift is proved modulo q .¹ The product identity reduces the exponent modulo q using $c_\alpha^q = 1$.²

References

- [DICCSPG17] Gemma De las Cuevas, J. Ignacio Cirac, Norbert Schuch, and David Pérez-García. Irreducible forms of matrix product states: Theory and applications. *Journal of Mathematical Physics*, 58(12):121901, 2017.

¹Formal declaration: `Fin.stepOrbitEquiv_add_one`, in `TNLean/Algebra/FinStepOrbit.lean`.

²Formal declaration: `Fin.exists_stepOrbit_phases`, in `TNLean/Algebra/FinStepOrbit.lean`.