Phase-Normalized Weights in the Normal-Canonical Implication to Irreducible Form

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Verdict: clarification (resolved).

1 The assertion

The periodic-MPS paper states that normal tensors are precisely periodic tensors of period 1, and that when all blocks in the irreducible decomposition are normal the decomposition is the canonical form in the sense of Ci15.[1] In the same paragraph the irreducible-form weights are fixed by the convention $\mu_j > 0$.

The blueprint records the corresponding implication: a normal canonical block family assembles to an irreducible-form tensor whose periods are all equal to 1.

2 The formal statements

The formal predicate for irreducible form records the paper's positive-weight convention: $\operatorname{Re}\mu_j > 0$ and $\operatorname{Im}\mu_j = 0$. The formal predicate for normal canonical block hypotheses, inherited from the non-periodic canonical-form formulation, assumes only that the complex weights are nonzero and ordered by modulus.

The paper introduces a minimal non-repeated basis of periodic tensors later in Section II. That basis condition is separate from the irreducible-form definition considered here.

The formalization now proves the phase-normalized construction. From a normal canonical block family with nonzero complex weights μ_j, it replaces μ_j by $|\mu_j|$ and A_j by $(\mu_j/|\mu_j|)A_j$. The assembled tensor has the same

[1] [DlCCSPG17, Section II]; the local source passage is arXiv:1708.00029, lines 258–271.

matrix-product-vector family, the irreducible-form weights are positive real numbers, and all periods are 1.[2]

3 Conclusion

This note is now a record of the resolved discrepancy. The paper's irreducible-form convention uses positive weights, and the formalization supplies the phase-normalization step needed to pass from the local normal-canonical predicate, whose weights are nonzero complex numbers, to that positive-weight convention without changing the represented matrix-product-vector family.

References

[DlCCSPG17] Gemma De las Cuevas, J. Ignacio Cirac, Norbert Schuch, and David Pérez-García. Irreducible forms of matrix product states: Theory and applications. *Journal of Mathematical Physics*, 58(12):121901, 2017.

[2]Formal declarations: `MPSTensor.toIsIrreducibleFormOfPhaseNormalized` and `MPSTensor.toIsIrreducibleFormOfPhaseNormalized_period_eq_one` in `TNLean/MPS/Periodic/NormalCanonicalPeriodOne.lean`.