

Peripheral Roots after Blocking a Periodic Tensor

2026-09-30

Verdict: local-correction (resolved).

1 The printed expression

The proof of Lemma 6 in [DICCSPG17], at `Papers/1708.00029/main.tex:451`, writes the peripheral spectrum of $\mathcal{E}_A^p$ as $\{\omega^l : 0 \leq l < m/r\}$, where $\omega = e^{2\pi i/m}$ and $r = \gcd(m, p)$. The exponent is missing the factor p . Throughout this calculation $m, p > 0$.

2 Correction

Put $q = m/r$. Spectral mapping gives

$$\text{spec}_{\text{per}}(\mathcal{E}_A^p) = \{\lambda^p : \lambda^m = 1\} = \{\omega^{pl} : 0 \leq l < q\} = \{z \in \mathbb{C} : z^q = 1\}. \quad (1)$$

In (1), ω^p has order $m/\gcd(m, p)$. For example, when $m = 4$ and $p = 2$, the printed set is $\{1, i\}$ whereas the correct set is $\{1, -1\}$. This is a correction to the displayed description of the roots, not a change to the period m/r asserted in the lemma.

3 Formal statement

The spectral mapping identity holds for every tensor and every positive blocking length.¹ For a periodic tensor of period m , the peripheral spectrum of $\mathcal{E}_A^p$ is exactly the set of q -th roots of unity.² The same line asserts that each compressed block C_α has the printed root set as its peripheral spectrum; with the same correction, that spectrum is again the set of q -th roots of unity.³ The full decomposition derives the cyclic projections, constructs the $\gcd(m, p)$ nonzero

¹Formal declaration: `MPSTensor.peripheralEigenvalues_blockTensor`, in `TNLean/MPS/Periodic/BlockingSpectrum.lean`.

²Formal declaration: `MPSTensor.IsPeriodic.peripheral_blockTensor`, in `TNLean/MPS/Periodic/BlockingSpectrum.lean`.

³Formal declaration: `MPSTensor.IsPeriodic.peripheral_compressed_stepOrbit_eq`, in `TNLean/MPS/Periodic/BlockingEigenvalues.lean`.

left-canonical compressed blocks, and, for $p > 0$, proves irreducibility and exact period $m/\gcd(m, p)$ for each; the isometries, total dimension, and equality of matrix product vectors are retained.⁴

References

- [DICCSPG17] Gemma De las Cuevas, J. Ignacio Cirac, Norbert Schuch, and David Pérez-García. Irreducible forms of matrix product states: Theory and applications. *Journal of Mathematical Physics*, 58(12):121901, 2017.

⁴Formal declaration: `MPSTensor.IsPeriodic.exists_stepOrbit_blockDecomposition`, in `TNLean/MPS/Periodic/PrescribedBlocking.lean`.