The Index Typo in the Pure-State Renormalization-Flow Equation

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Verdict: local-correction (resolved).

1 The malformed display

Theorem 3.1 of [CPGSV16] characterizes the limits of the pure-state renormalization flow. Its displayed equation at lines 400–403 is printed as

$$A^{i_1} A^{i_2} = \sum_{i_1, i_2, j} U_{(i_1, i_2), j} A^{j_1}.$$

The indices i_1, i_2 are free on the left but are summed on the right, and the index j_1 has not been introduced. Thus the display is not a well-formed component identity.

2 Intended equation

The renormalization step immediately preceding the theorem, at lines 389–394, blocks two physical indices and writes the resulting matrices as a physical isometry applied to a one-site family. With a common physical dimension at a fixed point, this gives

$$A^{i_1} A^{i_2} = \sum_{j} U_{(i_1, i_2), j} A^{j}$$

for every i_1, i_2. The two diagrams at lines 407–410 express the same contraction: two fixed external physical legs enter U, while only the one-site index j is summed. The Appendix proof at lines 1205–1209 also identifies this equation through equality of the completely positive maps $\mathcal{E}^2 = \mathcal{E}$, followed by physical Kraus isometry.

3 Formal treatment

The formal theorem uses the well-formed equation above. This is a local correction of the summation and output indices in the printed display. It changes neither the hypothesis that the tensor is a limit of the renormalization flow nor the conclusion that the blocked tensor is related to the original tensor by a physical isometry.

References

[CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: Annals of Physics 378 (2017), 100–149, 2016.