Adjacent regions obstruct the unrestricted CID forward implication in Cirac–Perez-Garcia–Schuch–Verstraete 2017

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Verdict: false-source (resolved).

1 Source statement and the adjacent gap

Definition 3.3 of [CPGSV16] defines correlations independent of the distance by quantifying over observables on any two disjoint regions.[1] Disjointness permits *adjacent* regions: with $n_2 = n_1' + 1$ there is no free site between the two observables, and the allowed shifts $n_1' - n_2 + 1 < \Delta < N - n_2'$ are exactly $\Delta \geq 1$, each leaving at least one free site on each complementary arc.

Expectation values are computed by the two-observable trace formula[2]

$$\langle V|O_1 O_2|V\rangle = \mathrm{tr}\big(\mathbb{E}_{O_1}\mathbb{E}^{\,n_2-n_1-1}\mathbb{E}_{O_2}\mathbb{E}^{\,N+n_1-n_2-1}\big)\,, \tag{1}$$

printed for single-site observables at positions $n_1 < n_2$. Generalized to block observables on the regions $[n_1, n_1']$ and $[n_2, n_2']$ of Definition 3.3, the exponents read $n_2 - n_1' - 1$ and $N + n_1 - n_2' - 1$. The forward implication "if $\mathbb{E}^2 = \mathbb{E}$, then one will have CID" (line 498) substitutes $\mathbb{E}^n = \mathbb{E}$ into this formula. Idempotence justifies that substitution only for $n \geq 1$. Between adjacent observables the middle exponent is $n_2 - n_1' - 1 = 0$, so the corresponding transfer block is $\mathbb{E}^0 = 1$, which idempotence does not identify with $\mathbb{E}$. The printed argument therefore covers only configurations with at least one free site between the observables, while the definition also quantifies over adjacent regions.

2 Counterexample: the Bell-pair chain

Take the canonical normal-tensor renormalization fixed point of `lem:charact-NT-pure-RFP` (line 1300) with trivial gauge $X = \mathbb{1}$, trivial isometry $U = \mathbb{1}$,

[1] Local source: `Papers/1606.00608/MPDO-22-12-17-2.tex:437--448`, label `ZCL`.
[2] Local source: `Papers/1606.00608/MPDO-22-12-17-2.tex:490--496`, label `Corr`.

and maximally mixed spectrum $\Lambda = (1/2, 1/2)$:

$$A^{(\alpha,\beta)}_{\gamma,\delta} = \delta_{\gamma,\alpha}\delta_{\delta,\beta}\sqrt{\tfrac{1}{2}}, \qquad (\alpha,\beta) \in \{0,1\}^2. \tag{2}$$

The physical index is the pair of spins at one node. The generated states are the chains of `eq:III_MPSRFP`–`eq:III_varphi` (lines 564–578): each node n carries two spins a_n, b_n, and the maximally entangled pair $|\varphi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ is shared between b_n and a_{n+1}.

The transfer map is

$$\mathbb{E}(X) = \operatorname{tr}(X)\operatorname{diag}(\Lambda) = \tfrac{1}{2}\operatorname{tr}(X)\mathbb{1}, \tag{3}$$

which is idempotent because $\operatorname{tr}\operatorname{diag}(\Lambda) = 1$. The tensor is a single normal tensor—it is already 1-block injective and left-canonical—so no repeated copies or copy weights are involved.

Now put the Pauli matrix Z on spin b_1 and Z on spin a_2. These are adjacent regions with zero gap, and the two-region expectation is the Bell-pair value

$$\operatorname{tr}\big(\mathbb{E}_{I\otimes Z}\mathbb{E}^0\mathbb{E}_{Z\otimes I}\mathbb{E}^2\big) = \langle\varphi|Z\otimes Z|\varphi\rangle = 1. \tag{4}$$

After the allowed shift $\Delta = 1$, which places one free site on each complementary arc, the expectation factorizes through the trace-form fixed point:

$$\operatorname{tr}\big(\mathbb{E}_{I\otimes Z}\mathbb{E}^1\mathbb{E}_{Z\otimes I}\mathbb{E}^1\big) = \langle\varphi|Z\otimes\mathbb{1}|\varphi\rangle^2 = 0. \tag{5}$$

The two values differ, so the tensor does not have CID in the unrestricted sense of the source definition, even though $\mathbb{E}^2 = \mathbb{E}$. All four assertions—idempotence, normality, the adjacent value 1, and the shifted value 0—have kernel-checked proofs.[3]

3 Consequence for the theorem surface

The unrestricted forward implication of line 498 is false as stated, and the counterexample is a single normal renormalization fixed point: unlike the raw-weight counterexample of `cpsv16_pure_zcl_raw_weight_counterexample.tex`, which refutes the converse direction of Theorem 3.8 (labelled `TheoremZCLPure`) using copy weights, no repeated copies are involved here. A repair that only normalizes copy weights therefore cannot save the forward direction; the quantification over adjacent regions must be removed instead.

The faithful repair is the positive-gap restriction, requiring at least one free site on each complementary arc. That restricted statement is proved: transfer idempotence implies physical CID when both complementary gaps are positive, formalized as `MPSTensor.isPositiveGapPhysicalCID_of_isTransferIdempotent`. The Bell-pair chain is consistent with the repair: its gap-positive

[3]Formal conjunction: `MPSTensor.bellPairChain_isNormalTensor_isTransferIdempotent_and_not_isPhysicalCID` in `TNLean/MPS/RFP/BellPairCIDObstruction`.

configurations do factorize, as the shifted value above shows. The source-faithful unrestricted predicate `MPSTensor.IsPhysicalCID` is retained with a docstring pointer to this note, and the gap recorded here closes the question left open in the raw-weight note, whose converse-direction counterexample did not settle the forward implication for adjacent physical regions.

The verdict is a source gap with a proved repair: the printed forward argument has an adjacent-region hole at $\mathbb{E}^0 = 1$, and the positive-gap statement is the correct replacement target for GitHub issue #4772.

References

[CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: Annals of Physics 378 (2017), 100–149, 2016.