Projective Reading of Proportional MPVs in CPSV16

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Verdict: clarification (resolved).

1 Source phrase

Cirac–Pérez-García–Schuch–Verstraete state Theorem `thm1` in [CPGSV16, lines 1167–1170], for tensors whose matrix-product vectors are "proportional to each other". The displayed source statement does not spell out whether the proportionality scalar at a fixed length is required to be nonzero. The two formal readings are

$$\exists c_N, V^{(N)}(A) = c_N V^{(N)}(B), \tag{1}$$

$$\exists c_N \neq 0, V^{(N)}(A) = c_N V^{(N)}(B). \tag{2}$$

$$V^{(N)}(A) = 0 \cdot V^{(N)}(B). \tag{3}$$

The scalar multiple with coefficient 0 gives no projective point.

2 Formal reading

The formal predicate $\texttt{NonzeroProportionalMPV}_2$ requires, at each positive length $N \geq 1$, a scalar $c_N \neq 0$ such that

$$V^{(N)}(A) = c_N V^{(N)}(B). \tag{4}$$

This is the projective reading of proportionality: the two vectors determine the same point in projective space. It is the reading used by the proof of Theorem `thm1`. The empty word is excluded because it is not a physical chain and its coefficient records only the bond-space dimension. In the block-selection step, the paper assumes that all overlaps of a fixed block on one side with the blocks on the other side tend to zero, expands the two BNT decompositions, and invokes Corollary `Lem1` to contradict proportionality. If c_N were allowed to be zero at a length where the vector on the left also vanished, the identity

$V^{(N)}(A) = 0 \cdot V^{(N)}(B)$ would not express equality of projective MPV families and would not supply the nonzero scalar used in the contradiction.

To make this last use explicit, write $a_j(N) = V^{(N)}(A_j)$ and $b_\ell(N) = V^{(N)}(B_\ell)$ for the BNT-sector vectors at length N. Let g_A and g_B be the numbers of sectors in the two decompositions, and let $\mu_j, \nu_\ell \in \mathbb{C}$ be the corresponding sector weights in the one-representative form used in the block-selection argument. The proportionality relation has the form

$$\sum_{j=1}^{g_A} \mu_j^N a_j(N) = c_N \sum_{\ell=1}^{g_B} \nu_\ell^N b_\ell(N). \tag{5}$$

Projecting this equality to a selected B-sector k gives

$$c_N \nu_k^N \langle b_k(N), b_k(N) \rangle = 0. \tag{6}$$

The contradiction is therefore the incompatibility of this equality with

$$c_N \neq 0, \qquad \nu_k \neq 0, \qquad \langle b_k(N), b_k(N) \rangle \neq 0. \tag{7}$$

Under the canonical-form hypotheses used in the source argument, the selected BNT block states have nonzero norm on the tail of lengths used in the contradiction; in that range the weak scalar-multiple predicate already forces the proportionality scalar to be nonzero. Thus the formal predicate records the projective form needed at the use site, rather than adding a separate asymptotic assumption to the block-selection argument.

3 Effect on the theorem

The nonzero condition does not change the intended conclusion of the source theorem. It records the conventional mathematical meaning of proportional nonzero vectors in the source argument. The weaker scalar-multiple predicate `ProportionalMPV`$_2$, where c_N may vanish, remains available for older auxiliary lemmas and is likewise indexed only by positive lengths. It is not used as the paper-faithful hypothesis for the proportional Fundamental Theorem.

4 Elimination plan

No extra mathematical theorem is needed for the present projective reading of CPSV16 Theorem `thm1` [CPGSV16, lines 1167–1170]. If a literal weak-scalar reading is later required, it should be formalized as a separate bridge theorem: prove from the source canonical-form hypotheses that the relevant MPV vectors do not vanish at the lengths used in the argument, then derive `NonzeroProportionalMPV`$_2$ from the weak predicate before applying the block-selection proof.

References

[CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: Annals of Physics 378 (2017), 100–149, 2016.