Normalization of the tensor printed in CPSV16 Example 4.12

2026-08-10

Verdict: false-source (open).

1 Printed representative

Example 4.12 of Cirac–Perez-Garcia–Schuch–Verstraete (`Papers/1606.00608/MPDO-22-12-17-2.tex:932--939`) prints a tensor M with $d = D = 2$ and

$$1 = M^{00}_{00} = M^{11}_{00} = M^{00}_{11} = -M^{11}_{11},$$

all other components being zero. In matrix notation this says

$$M^{00} = I, \qquad M^{11} = \sigma_z, \qquad M^{01} = M^{10} = 0,$$

where $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$. Closing the virtual indices gives exactly the displayed family

$$\rho^{(N)}(M) = I^{\otimes N} + \sigma_z^{\otimes N} \qquad (N > 0).$$

Its trace is 2^N, not 1. Thus the printed M is an unnormalized tensor representative even though the resulting positive operator determines a normalized density operator after division by 2^N.

2 The fixed-representative mismatch

The physical-trace transfer of the printed tensor is

$$\mathcal{T}_M = M^{00} + M^{11} = I + \sigma_z = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}.$$

Consequently

$$\mathcal{T}_M^2 = 2\mathcal{T}_M \qquad \text{but} \qquad \mathcal{T}_M^2 \neq \mathcal{T}_M.$$

This proves the project's scale-invariant relation with scale 2, but it does not prove the literal ZCL equation of Definition 4.2. Indeed, that equation fails. Moreover, the trace-preserving maps $\mathcal{S}$ and $\mathcal{T}$ in Definition 4.1 force equality of the one-site and two-site traces for every virtual boundary. Equivalently, they force literal idempotence $\mathcal{T}_M^2 = \mathcal{T}_M$. The printed representative therefore cannot satisfy that fixed-tensor renormalization condition.

This does not contradict the normalized density family. Scaling the local tensor by $1/2$ gives the separate representative

$$\widehat{M} = \tfrac{1}{2}M, \qquad \mathcal{T}_{\widehat{M}} = \tfrac{1}{2}\mathcal{T}_M, \qquad \mathcal{T}_{\widehat{M}}^2 = \mathcal{T}_{\widehat{M}}.$$

Moreover,

$$\rho^{(N)}(\widehat{M}) = 2^{-N}\big(I^{\otimes N} + \sigma_z^{\otimes N}\big),$$

which is precisely the normalized density family. This makes $\widehat{M} = (1/2)M$ the likely representative intended by the sentence on line 938 asserting the existence of trace-preserving maps. The paper does not print this factor in the tensor components or explicitly change representatives before that sentence.

3 Formalization boundary

The current formalization keeps the two representatives separate. For the literal printed M it proves the exact positive-length formula, positivity, one-site loss of the σ_z term, saturation of the area law, source zero correlation length only in the project's scale-invariant sense, explicit failure of literal ZCL, and hence failure of the fixed-representative trace-preserving renormalization condition.

For $\widehat{M} = (1/2)M$, let $\pi(a,b) = a \oplus b$ and define Kraus operators

$$C_{a,b} = |\pi(a,b)\rangle\langle a,b|, \qquad R_{a,b} = 2^{-1/2}|a,b\rangle\langle\pi(a,b)|.$$

Each parity fibre has two elements, and hence

$$\sum_{a,b} C_{a,b}^\dagger C_{a,b} = I_4, \qquad \sum_{a,b} R_{a,b}^\dagger R_{a,b} = I_2.$$

The corresponding completely positive maps are trace preserving. Since

$$\widehat{M}^{aa}\widehat{M}^{bb} = \tfrac{1}{2}\widehat{M}^{\pi(a,b)\pi(a,b)}$$

and the off-diagonal physical letters vanish, direct substitution proves both channel equations of Definition 4.1 for every virtual boundary matrix. These results are recorded by `MPOTensor.CPSVExample412NormalizedRFP.Mhat`, `MPOTensor.CPSVExample412NormalizedRFP.mpo_Mhat_eq_normalizedMPO_M`, `MPOTensor.CPSVExample412NormalizedRFP.Mhat_isMPDO`, and `MPOTensor.CPSVExample412NormalizedRFP.Mhat_isRFPViaTS`.

This conclusion concerns precisely the two channel equations. It does not settle a second normalization tension in the source. As a doubled-index MPS, M has two bond-one sectors with physical vectors $(1,0,0,1)$ and $(1,0,0,-1)$. Their normal representatives are obtained by division by $\sqrt{2}$. Consequently the two horizontal normal-block weights of $\widehat{M}$ are both $1/\sqrt{2}$. Neither has modulus one, whereas line 246 assumes that at least one weight does. Thus the proved bare channel predicate must not be presented as a proof that $\widehat{M}$ obeys every standing canonical-form convention in Definition 4.1. This boundary is also recorded in `docs/paper-gaps/cpsv16_unit_weight_rfp_scale_tension.tex`.

The independent four-site entropy calculation is also complete. For the normalized state, regroup the sites as $A = \{0\}$, $B = \{1, 3\}$, and $C = \{2\}$. The AB and BC marginals are $I_8/8$, the B marginal is $I_4/4$, and the full state is uniform on eight even-parity configurations. Thus

$$S(ABC) = 3 \log 2, \qquad S(B) = 2 \log 2, \qquad S(AB) = S(BC) = 3 \log 2,$$

so equality in strong subadditivity fails by the strictly positive defect $\log 2$. This proves the state-specific Markov obstruction without using the normalized fixed-point maps.

The full GSNNCH exclusion is already a statement about the normalized family because `MPOTensor.IsGSNNCH` quantifies over `MPOTensor.normalizedMPO`. It is now proved directly for the printed tensor. At four sites, the sector sum in Definition 4.8 splits into positive factors built from the complementary adjacent-bond products $B_3^{(x)} B_0^{(x)}$ and $B_1^{(x)} B_2^{(x)}$, acting respectively on the three-site windows $(3, 0, 1)$ and $(1, 2, 3)$. Orthogonality of distinct sectors removes the cross terms in their product, while commutativity within one sector identifies the product of the two factors with the four-bond sector product. The resulting positive overlapping factors commute and give the quantum Markov decomposition `MPOTensor.nonempty_quantumMarkovDecomposition_fourCycle_of_isGSNNCHAt`, hence the strong-subadditivity equality `MPOTensor.isSSAEquality_fourCycle_of_isGSNNCHAt`. This derivation uses only Definition 4.8 and the Beigi–Hayden–Jozsa–Petz–Winter structure; it does not assume zero correlation length, normality, injectivity, primitivity, a single sector, or completeness of the sector projections.

The identity `MPOTensor.CPSVExample412Literal.fourCycleTripartiteState_eq_generic` shows that this general regrouping is exactly the state used in the literal entropy calculation. Consequently `MPOTensor.CPSVExample412Literal.M_not_isGSNNCH` proves $\neg\,\mathrm{IsGSNNCH}(M)$. This discharges the missing and formerly unlinked four-cycle task tracked in GitHub issue #6072 and the missing Example 4.12 conclusion tracked in GitHub issue #6045. The earlier attempt to pass through Proposition *prop4to2* had proof-path drift: that proposition belongs to a different condition-level implication and is not used here. Transport to the separately named $\widehat{M}$ is unnecessary because the GSNNCH predicate already uses the normalized family.

4 Verdict

Verdict: false as printed for the literal fixed representative. The tensor M printed in Example 4.12 is the formal tensor `MPOTensor.CPSVExample412Literal.M`. Its positive-length operator formula is proved by `MPOTensor.CPSVExample412Literal.rho_eq_finKronecker`, and its saturation statement by `MPOTensor.CPSVExample412Literal.M_isSAL`. The transfer calculation `MPOTensor.CPSVExample412Literal.physTraceTransfer_M_sq` has scale 2, while `MPOTensor.CPSVExample412Literal.physTraceTransfer_M_not_idempotent` proves failure of literal idempotence. Therefore `MPOTensor.CPSVExample412Literal.M_not_isRFPViaTS` proves that the fixed-representative RFP assertion is false for the tensor exactly as printed.

The normalized tensor $\widehat{M} = (1/2)M$ is the likely local correction. Its periodic family is identified with the normalized periodic family of M by tensor-level scalar rescaling and `MPOTensor.mpo_smul`; it is not an independent tensor presentation. The parity Kraus families above prove the bare trace-preserving channel predicate without additional hypotheses, discharging the mathematical task tracked in GitHub issue #6044. They do not repair the incompatible line-246 unit-weight convention, so the formal verdict must remain separated into the false literal claim, the true normalized channel equations, the unresolved unit-weight boundary, and the now-proved full GSNNCH exclusion.

For the normalized four-site state, `MPOTensor.CPSVExample412Literal.fourCycleTripartiteState_not_isSSAEquality` proves the strict strong-subadditivity defect. Its proof uses the exact marginals `MPOTensor.CPSVExample412Literal.traceC_fourCycleTripartiteState`, `MPOTensor.CPSVExample412Literal.traceA_fourCycleTripartiteState`, and `MPOTensor.CPSVExample412Literal.traceAC_fourCycleTripartiteState`, together with their entropy calculations, discharging the mathematical task tracked in GitHub issue #6073. The general four-cycle Markov implication and the exact coordinate identity then give `MPOTensor.CPSVExample412Literal.M_not_isGSNNCH`, discharging the tasks tracked in GitHub issue #6072 and GitHub issue #6045 without hypotheses beyond the source GSNNCH definition.