

Exact arithmetic scope for CPSV16 Examples 4.10 and 4.11

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Verdict: scope-restriction (open).

1 Source statements and derived comparisons

Example 4.10 of Cirac–Perez-Garcia–Schuch–Verstraete (`Papers/1606.00608/MPD0-22-12-17-2.tex:897--905`) prints a channel with a repeated left-qubit label, four decimal entropy values, and the claim that the mutual information does not saturate except at the stated exceptional probabilities. It prints no exact ratio or integer comparison. Replacing the repeated label by the network-consistent left-right correlated flip and setting the flip probability to $p = 1/4$ reproduces the decimal entropies and gives the independently derived comparison

$$2^{32}7^7 > 3^35^{20}.$$

The corrected channel, reduced spectra, and logarithmic reduction are recorded in `docs/paper-gaps/cpsv16_examples_4_10_4_11_entropy.tex`, not in the source paper.

Lines 907–924 print an arbitrary-length family for Example 4.11, its diagonal bond weights, and the claim that it satisfies saturation of the area law but not zero correlation length. They contain no four-site specialization, reduced spectra, entropy values, or exact integer comparison.

Independently specializing the printed Example 4.11 family to four sites gives

$$41^{82}5^{50} > 2^{48}13^{52}7^{112}.$$

The finite-ring distribution, reduced spectra, and logarithmic reduction that yield this comparison are recorded in `docs/paper-gaps/cpsv16_examples_4_10_4_11_entropy.tex`; they are not statements printed in the source paper. Both sides of each comparison are positive integers, so each strict inequality is equivalent to positivity of the real logarithm of the corresponding ratio.

2 Certified statements

The module `TNLean/MPS/MPDO/CPSVExamples410411Arithmetic` machine-checks the two integer inequalities, `CPSVExamples410411Arithmetic.example410_integer_inequality` and `CPSVExamples410411Arithmetic.example411_integer_inequality`, and the positivity of the corresponding logarithmic ratios, `CPSVExamples410411Arithmetic.example410_log_ratio_pos` and `CPSVExamples410411Arithmetic.example411_log_ratio_pos`.

3 Scope boundary

For the four-site states, let S_L denote the entropy of L neighboring sites and define the chain mutual information by $I_L = S_L + S_{4-L} - S_4$. These are scalar arithmetic statements. They do not identify the reduced-state spectra of the printed tensors, do not assert the entropy comparisons $I_2 > I_1$ discussed in Examples 4.10 and 4.11, and do not assert saturation or failure of an area law. Deriving the entropy comparisons requires separate formal links from the printed tensors to the corresponding entropy expressions. Those links, and the resulting corrections to or confirmations of the printed zero-correlation-length and SAL claims, remain under the Example 4.10 and Example 4.11 implementation tasks GitHub issue #6301 and GitHub issue #6302, respectively.

4 Verdict

Verdict: scope restriction. The arithmetic endpoints are certified, but the entropy conclusions are not. The exact comparisons on which the four-site calculations rest are machine-checked, but nothing here is a formalization of the printed entropy or area-law claims themselves.