A Phase Obstruction to Canonical-Form Renormalization-Flow Convergence

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Verdict: false-source (resolved).

1 The source assertion

Around Theorem 3.1, [CPGSV16, lines 382 and 417] asserts that the renormalization flow starting from a tensor in canonical form always converges. Appendix B identifies one renormalization step with $\mathcal{E} \mapsto \mathcal{E}^2$ at lines 1205–1209 and gives the canonical-form transfer decomposition at lines 1211–1243. The conclusion at line 1244 is that the normalization

$$|\mu_{j,q}| \leq 1$$

implies convergence of the transfer powers.

This conclusion overlooks relative phases between repeated copies of the same normal tensor. In the decomposition at lines 1220–1226, the corresponding summands carry the factors

$$\mu_{j,q}\overline{\mu_{j,q'}}\,\mathbb{E}_{j,j}.$$

When both weights have modulus one, normalization does not force the relative phase to converge under repeated squaring.

2 Phase-oscillation counterexample

Let

$$\omega = e^{2\pi i/3}$$

and consider the tensor with one physical component and bond dimension two given by

$$A^1 = \begin{pmatrix} 1 & 0 \\ 0 & \omega \end{pmatrix}.$$

It is in the literal canonical form of [CPGSV16, lines 238–246]: its two one-dimensional blocks are normal tensors with normalized weights 1 and ω.

For the matrix unit e_{21}, the transfer map satisfies

$$\mathcal{E}_A(e_{21}) = A^1 e_{21} (A^1)^\dagger = \omega e_{21}.$$

Consequently,

$$\mathcal{E}_A^{2^n}(e_{21}) = \omega^{2^n} e_{21}.$$

The residue of 2^n modulo 3 alternates between 1 and 2. Hence this sequence alternates between ωe_{21} and $\omega^2 e_{21}$, and the dyadic renormalization flow does not converge. The unrestricted sequence $\mathcal{E}_A^N$ is likewise three-periodic.

3 Faithful boundary

Theorem 3.1 itself is unaffected: it characterizes tensor classes that *do* appear as limits, and does not assert that every canonical-form initial tensor has a limit. Primitive-transfer convergence for each individual normal block also remains valid.

For the full repeated-copy canonical form, convergence requires control of the relative phases. A sufficient condition is that all unit-modulus weights belonging to one normal tensor have the same phase. More precisely, dyadic convergence requires each sequence

$$(\mu_{j,q}\overline{\mu_{j,q'}})^{2^n}$$

arising from two unit-modulus repeated-copy weights to converge. Without such a condition, the universal convergence assertion at lines 1211–1244 is refuted rather than an unproved formalization target.

References

[CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: Annals of Physics 378 (2017), 100–149, 2016.