Active Canonical-Form Blocks in the BNT Characterization

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Verdict: local-correction (resolved).

1 The two source formulations

Cirac–Pérez-García–Schuch–Verstraete construct the canonical form by listing the invariant blocks of the completely positive map and normalizing each block separately [CPGSV16, lines 214–225]. Thus the blocks produced by the construction are precisely the blocks which occur in the tensor. The subsequent displayed definition [CPGSV16, lines 238–244], however, writes

$$A^i = \bigoplus_{k=1}^{r} \mu_k A_k^i \qquad \texttt{(eq:II\_CF1)}$$

without explicitly requiring every μ_k to be nonzero. Read literally, this display also permits a listed block with zero weight. Such a block contributes nothing to any positive-length matrix product vector.

The source definition of a basis of normal tensors [CPGSV16, lines 271–274] requires, as its first condition, that every candidate representative A_j is normal. Proposition 2.7 then characterizes the basis by coverage of the canonical-form blocks and pairwise gauge-phase distinction [CPGSV16, lines 278–280 and 1135–1146]. Its displayed right-hand side does not repeat candidate normality, because that condition is already part of the term "basis of normal tensors."

2 The local correction

To state an equivalence which is valid under the literal displayed canonical-form hypothesis, let

$$K_{\mathrm{act}} = \{k : \mu_k \neq 0\}. \tag{1}$$

Coverage is required exactly for $k \in K_{\mathrm{act}}$. Removing the other indices leaves every positive-length matrix product vector unchanged, since $\mu_k^N = 0$ whenever

$N > 0$ and $\mu_k = 0$. The active family has nonzero weights and therefore satisfies the hypothesis needed by the source phase-class argument.

Candidate normality is placed explicitly on the characterized side of the equivalence. The resulting statement says that a candidate family is a basis of normal tensors if and only if

1. every candidate representative is normal;
2. every active canonical-form block is gauge-phase equivalent to a candidate representative; and
3. distinct candidate representatives are not gauge-phase equivalent.

The third condition is the source notion of minimality. No converse coverage condition is added: Definition 2.4 permits an additional normal representative whose coefficient is identically zero, provided the full candidate family is eventually linearly independent.

This correction is recorded in the formal characterization.[1] It reconciles the literal canonical-form display with the constructive nonzero-block reading and makes the normality condition from Definition 2.4 visible in the equivalence. It does not strengthen or weaken the mathematical characterization of the active canonical-form blocks.

References

[CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: Annals of Physics 378 (2017), 100–149, 2016.

[1] Formal declaration: `MPSTensor.isCPSVBasisOfNormalTensors_iff_canonicalForm_covered_and_minimal`.