

Critical Parameters and Boundary Localization in the One-Species PVBS Chain

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Verdict: false-source (resolved).

1 The assertion

The review arXiv:2011.12127, Section IV.C.4 (HTML Section IV.3.4), gives the matrices $A^0 = \text{diag}(1 + \lambda, 1)$ and $A^1 = |0\rangle\langle 1|$, with $\lambda \in [-1, 1]$. It describes a two-dimensional open-chain ground space, a boundary-localized particle, and a unique product vacuum on the periodic chain. The printed right-edge condition is $\lambda > 1$.

2 The point at issue

Write $q = 1 + \lambda$. A particle at the zero-indexed site k has amplitude q^k . Thus left localization occurs for $0 \leq q < 1$ and right localization for $q > 1$: in the review's parameter these are $\lambda < 0$ and $\lambda > 0$. The printed right-edge condition is outside its stated parameter interval. At $q = 1$, the particle is the delocalized W vector. On a two-site ring, $|10\rangle + |01\rangle$ satisfies both local constraints and is independent of the vacuum. Therefore unconditional periodic vacuum uniqueness is false.

3 The corrected statement

For any complex q , the open parent Hamiltonian with two-site interactions has ground space

$$\text{span} \left\{ |0 \cdots 0\rangle, \sum_{k=0}^{N-1} q^k |0 \cdots 1_k \cdots 0\rangle \right\}, \quad N \geq 2. \quad (1)$$

Its dimension is two, including $q = 0$ and $q = 1$. On a periodic chain, the vacuum is the unique ground state whenever $q^N \neq 1$. In particular, this holds at every length $N \geq 2$ if $|q| \neq 1$. The latter condition is sufficient, not necessary at any fixed length. To prove uniqueness, write any periodic ground vector as

vacuum plus b times the open-chain particle. A one-site translation remains a ground vector. Comparing the first two particle amplitudes forces $b = bq^N$, hence $b = 0$. The vacuum satisfies every local constraint.

For $|q| < 1$, the left-edge amplitudes decay as $|q|^k$. For $|q| > 1$, the amplitude k sites from the right edge, divided by the rightmost amplitude, is q^{-k} . These describe localization without asserting a uniform spectral gap.

4 Source conventions and conclusion

Bachmann and Nachtergaele, arXiv:1112.4097, Section II, equations (1)–(8), use positive hopping parameters and phases, and explicitly exclude unit hopping in their boundary-localization discussion. Their matrix-product ordering differs from the left-to-right ordering above; the displayed one-particle amplitudes fix the convention. More explicitly, $q = \mu e^{i\theta}$ and the forbidden local ket is $|01\rangle - \bar{q}|10\rangle$, whose orthogonal constraint is $a_{01} = qa_{10}$. The primary source assumes $\mu > 0$; $q = 0$ is an algebraic endpoint extension. The formalization permits complex q , while the review’s real family is recovered by $q = 1 + \lambda$. The corrected finite-chain ground spaces and the critical two-site counterexample resolve the discrepancy. No spectral-gap theorem is claimed here.¹

¹Formal declarations: `MPSTensor.ker_openParentHamiltonianES_pvbs`, `MPSTensor.ker_parentHamiltonian_pvbs`, `MPSTensor.pvbs_critical_periodic_counterexample`, and `MPSTensor.pvbsEdge_right_decay`.