

Scope of the Physical-Deformation Ground-Space Correspondence Relative to CPGSV21

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Verdict: scope-restriction (open).

1 Source statement

Section IV.C of Cirac–Pérez-García–Schuch–Verstraete [1] considers an invertible map Λ acting on the physical space $\mathbb{C}^d$ of a tensor network state $|\psi\rangle$ defined on a general graph $G = (V, E)$. Under the action of $\Lambda^{\otimes V}$, the deformed state $|\psi'\rangle = \Lambda^{\otimes V}|\psi\rangle$ has deformed local parent interactions

$$h'_e = (\Lambda^{-1\dagger})^{\otimes \partial e} h_e (\Lambda^{-1})^{\otimes \partial e}, \quad (1)$$

where ∂e is the support of the interaction term. The source asserts that the map $\Lambda^{\otimes V}$ establishes a one-to-one correspondence between the ground spaces of the parent Hamiltonians of $|\psi\rangle$ and $|\psi'\rangle$.

2 Formalization scope

The formal development in `TNLean/MPS/ParentHamiltonian/PhysicalDeformation.lean` and Chapter 13 of the blueprint formalizes this correspondence specifically for one-dimensional matrix product states on a periodic chain with N sites, establishing

$$\mathcal{G}_{N,L}(\Lambda A) = \Lambda^{\otimes N} \mathcal{G}_{N,L}(A) \quad (2)$$

and $\ker H_N(\Lambda A) = \Lambda^{\otimes N} \ker H_N(A)$ for an invertible matrix $\Lambda \in M_d(\mathbb{C})$ in the finite-chain range $0 < N$ and $L \leq N$.¹ The general-graph formulation of Section IV.C of Ref. [1] is not formalized.

¹Formal declaration: `MPSTensor.ker_parentHamiltonian_rotatePhysical`.

3 Elimination plan

Closing the scope restriction requires formalizing tensor network states and their parent Hamiltonians on an arbitrary finite graph $G = (V, E)$, with interaction terms supported on neighbourhoods ∂e of edges $e \in E$. For an invertible on-site operator Λ , the corresponding graph-level physical deformation $\Lambda^{\otimes V}$ must be shown to transform each local interaction term according to (1) and to map the kernel intersection $\bigcap_{e \in E} \ker h_e$ onto $\bigcap_{e \in E} \ker h'_e$. The present periodic-chain theorem follows by specializing the general-graph statement to a cycle graph C_N with contiguous edge windows of length L .

References

- [1] J. I. Cirac, D. Pérez-García, N. Schuch, and F. Verstraete. Matrix product states and projected entangled pair states: Concepts, symmetries, theorems. *Rev. Mod. Phys.*, 93:045003, 2021.