

Zero-Weight Sector Completions in the MPDO Refinement Map

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Verdict: local-correction (resolved).

1 The undefined source quotient

In the proof of Proposition C.7, [CPGSV16, Appendix C.2, lines 1523–1535], the sectorwise refinement map is written as

$$\mathcal{T}_{k,h}(X) = \frac{1}{a_k b_h} X \otimes \left[\bigoplus_l (\eta_{k,l} \otimes \eta_{l,h}) \right].$$

The hypotheses of Proposition C.7 require

$$\eta_{k,h} \succeq 0, \quad \text{tr}(\eta_{k,h}) = a_k b_h, \quad \sum_k a_k b_k = 1,$$

but do not state that every product $a_k b_h$ is nonzero. Consequently the displayed quotient is not defined on a zero-weight sector pair.

2 Trace-preserving completion

Set

$$\Omega_{k,h} = \bigoplus_l (\eta_{k,l} \otimes \eta_{l,h}).$$

Positivity and the source trace calculation give

$$\Omega_{k,h} \succeq 0, \quad \text{tr}(\Omega_{k,h}) = a_k b_h.$$

Thus $a_k b_h = 0$ implies $\Omega_{k,h} = 0$. On an active sector pair one adjoins the density operator $(a_k b_h)^{-1} \Omega_{k,h}$. On a zero-weight pair one may adjoin any density operator on the same subspin space. This defines a trace-preserving completely positive map on every sector pair.

The choice on a zero-weight pair does not affect the refinement identity: the preceding partial trace $\mathcal{T}_0$ produces the factor $a_k b_h B_{k,h}(X)$, which is already zero on that pair. Equivalently, the corresponding block of the three-site closure is $B_{k,h}(X) \otimes \Omega_{k,h} = 0$.

3 Verdict

The inactive-sector density is a local completion of the map omitted from the source formula. It adds no hypothesis to Proposition C.7 and leaves its two-site to three-site identity unchanged. Formal declarations implementing this completion should carry a *Local fix (zero-weight quotient)* marker and cite this file.

References

- [CPGSV16] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product density operators: Renormalization fixed points and boundary theories. Preprint; published version: *Annals of Physics* 378 (2017), 100–149, 2016.